Math 241 – Quiz # 2b

Name: Solution Sr #: ____________

1. Let $B=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $C=\begin{bmatrix} -g & -h & -i \\ 5d & 5e & 5f \\ a & b & c \end{bmatrix}$. If $\det(B)=2$, find $\det(C)$?

$$\det(B)=\begin{vmatrix} -g & -h & -i \\ 5d & 5e & 5f \\ a & b & c \end{vmatrix} = 5\begin{vmatrix} -g & -h & -i \\ d & e & f \\ a & b & c \end{vmatrix} \xrightarrow{-R_1} = 5(-1)\begin{vmatrix} g & h & i \\ d & e & f \\ a & b & c \end{vmatrix} \xrightarrow{R_1 \leftrightarrow R_3}$$

$$= 5(-1)(-1)\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5(-1)(-1)\det(B) = 5(2) = 10$$

2. Find the values of k for which the matrix $B=\begin{bmatrix} 2 & -2 & 0 \\ 1 & 3 & 1 \\ 0 & 2 & k \end{bmatrix}$ is invertible.

B is invertible iff $\det(B) \neq 0$

$$\det(B)=\begin{vmatrix} 2 & -2 & 0 \\ 1 & 3 & 1 \\ 0 & 2 & k \end{vmatrix} = 2\begin{vmatrix} 3 & 1 \\ 2 & k \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 0 & k \end{vmatrix}$$

$$= 2(3k-2)+2k$$
$$= 6k-4+2k$$
$$= 8k-4$$

B is invertible $\iff 8k-4 \neq 0$

$\iff k \neq \frac{1}{2}$

So, B is invertible for all values of k except $k=\frac{1}{2}$.

3. Consider the following system of linear equations:

$$\begin{aligned} -2x_1 + 3x_2 - x_3 &= 1 \\ x_1 + 2x_2 - x_3 &= 4 \\ -2x_1 - x_2 + x_3 &= -3 \end{aligned}$$

Use Cramer's rule to solve for x_3?

The coefficients matrix $A = \begin{bmatrix} -2 & 3 & -1 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$

$$|A| = \begin{vmatrix} -2 & 3 & -1 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{vmatrix} = -2\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$$

$$= -2(1) - 3(-1) - 1(3) = -2 + 3 - 3 = \boxed{-2}$$

$$|A_3| = \begin{vmatrix} -2 & 3 & 1 \\ 1 & 2 & 4 \\ -2 & -1 & -3 \end{vmatrix} = -2\begin{vmatrix} 2 & 4 \\ -1 & -3 \end{vmatrix} - 3\begin{vmatrix} 1 & 4 \\ -2 & -3 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$$

$$= -2(-2) - 3(5) + 3$$

$$= 4 - 15 + 3 = \boxed{-8}$$

$$\therefore \quad x_3 = \frac{|A_3|}{|A|} = \frac{-8}{-2} = 4$$