Math 241 – Quiz # 2

Name: Solution

Sr #: ______________

1. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $B = \begin{bmatrix} -g & -h & -i \\ 4d & 4e & 4f \\ a & b & c \end{bmatrix}$. If $\det(A) = 6$, find $\det(B)$?

$$\det(B) = \begin{vmatrix} -g & -h & -i \\ 4d & 4e & 4f \\ a & b & c \end{vmatrix} = 4\begin{vmatrix} -g & -h & -i \\ d & e & f \\ a & b & c \end{vmatrix} \xrightarrow{-R_1} = 4(-1)\begin{vmatrix} g & h & i \\ d & e & f \\ a & b & c \end{vmatrix} \xrightarrow{R_1 \leftrightarrow R_3}$$

$$= 4(-1)(-1)\begin{vmatrix} a & b & c \\ d & e & f \\ a & b & c \end{vmatrix} = 4(-1)(-1)\det(A)$$

$$= 4(6)$$

$$= 24$$

2. Find the values of k for which the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 0 & 2 & K \end{bmatrix}$ is invertible.

A is invertible iff $\det(A) \neq 0$.

$$\det(A) = \begin{vmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 0 & 2 & k \end{vmatrix} = 1\begin{vmatrix} 3 & 1 \\ 2 & k \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 0 & k \end{vmatrix}$$

$$= 3k - 2 - 2k$$

$$= k - 2$$

A is invertible when $k - 2 \neq 0$

$k \neq 2$

So, A is invertible for all values of k except $k = 2$.

3. Consider the following system of linear equations:

$$-2x_1 + 3x_2 - x_3 = 1$$
$$x_1 + 2x_2 - x_3 = 4$$
$$-2x_1 - x_2 + x_3 = -3$$

Use Cramer's rule to solve for x_2?

The Coefficients matrix $A = \begin{bmatrix} -2 & 3 & -1 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix}$, constant: $B = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$

$$|A| = \begin{vmatrix} -2 & 3 & -1 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{vmatrix} = -2\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$$

$$= -2(1) - 3(-1) - 1(3) = -2+3-3 = \boxed{-2}$$

$$|A_2| = \begin{vmatrix} -2 & 1 & -1 \\ 1 & 4 & -1 \\ -2 & -3 & 1 \end{vmatrix} = -2\begin{vmatrix} 4 & -1 \\ -3 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ -2 & -3 \end{vmatrix}$$

$$= -2(1) - 1(-1) - 1(5)$$
$$= -2 + 1 - 5 = \boxed{-6}$$

$$x_2 = \frac{|A_2|}{|A|} = \frac{-6}{-2} = 3$$