HW#7-b

$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 4 \\ 1 & -1 & 5 \end{bmatrix}$. To find $\det(A)$.

Method 1 by <u>Cofactor expansion</u>: (along 2^{nd} row)

$$\begin{vmatrix} 3 & 1 & -1 \\ 2 & 0 & 4 \\ 1 & -1 & 5 \end{vmatrix} = -2\begin{vmatrix} 1 & -1 \\ -1 & 5 \end{vmatrix} + 0 - 4\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} = -2(4) - 4(-4) = 8$$

Method 2 by <u>arrows</u> technique:

$$\left|\begin{array}{ccc|cc} 3 & 1 & -1 & 3 & 1 \\ 2 & 0 & 4 & 2 & 0 \\ 1 & -1 & 5 & 1 & -1 \end{array}\right| = 0 + 4 + 2 - 0 + 12 - 10 = 8$$

Method 3 by <u>row reduction</u>:

$$\begin{vmatrix} 3 & 1 & -1 \\ 2 & 0 & 4 \\ 1 & -1 & 5 \end{vmatrix} \xrightarrow{R_1 \leftrightarrow R_2} = - \begin{vmatrix} 1 & -1 & 5 \\ 2 & 0 & 4 \\ 3 & 1 & -1 \end{vmatrix} \xrightarrow[-3R_1+R_3]{-2R_1+R_2} = - \begin{vmatrix} 1 & -1 & 5 \\ 0 & 2 & -6 \\ 0 & 4 & -16 \end{vmatrix} \xrightarrow{\frac{1}{2}R_2}$$

$$-2\begin{vmatrix} 1 & -1 & 5 \\ 0 & 1 & -3 \\ 0 & 4 & -16 \end{vmatrix} \xrightarrow{\frac{1}{4}R_3} = -2(4)\begin{vmatrix} 1 & -1 & 5 \\ 0 & 1 & -3 \\ 0 & 1 & -4 \end{vmatrix} \xrightarrow[-R_2+R_3]{R_2+R_1} = -2(4)\begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & -1 \end{vmatrix}$$

$$= -2(4)(1 \times 1 \times -1)$$

$$= 8$$