

HW # 6

1 a) $\det(A) = \sqrt{2}\sqrt{3} - 4\sqrt{6} = \sqrt{6} - 4\sqrt{6} = -3\sqrt{6}$

b) $\det(B) = -1 \begin{vmatrix} 3 & 1 \\ -3 & 5 \end{vmatrix} + 0 - (-4) \begin{vmatrix} 3 & 3 \\ 1 & -3 \end{vmatrix}$
 $= -1(18) + 4(-12) = -66$

2 $M_{23} = \begin{vmatrix} 3 & 3 \\ 1 & -3 \end{vmatrix} = -12$, $M_{33} = \begin{vmatrix} 3 & 3 \\ 1 & 0 \end{vmatrix} = -3$

$C_{23} = (-1)^5 M_{23} = 12$, $C_{33} = (-1)^6 M_{33} = -3$

3 a) $\det(H) = 4 \begin{vmatrix} -3 & 3 \\ 7 & 0 \end{vmatrix} - 5 \begin{vmatrix} 1 & 2 \\ 7 & 0 \end{vmatrix} + 6 \begin{vmatrix} 1 & 2 \\ -3 & 3 \end{vmatrix}$
 $= 4(-21) - 5(-14) + 6(9) = 40$

b) $\det(H) = -(-3) \begin{vmatrix} 2 & 4 \\ 0 & 6 \end{vmatrix} + 3 \begin{vmatrix} 1 & 4 \\ 7 & 6 \end{vmatrix} - 5 \begin{vmatrix} 1 & 2 \\ 7 & 0 \end{vmatrix}$
 $= 3(12) + 3(-22) - 5(-14) = 40$

c) $\begin{vmatrix} 1 & 2 & 4 & | & 1 & 2 \\ -3 & 3 & 5 & | & 3 & 3 \\ 7 & 0 & 6 & | & 7 & 0 \end{vmatrix}$

$\det(H) = 1 \times 3 \times 6 + 2 \times 5 \times 7 + 4 \times -3 \times 0 - 4 \times 3 \times 7 - 1 \times 5 \times 0 - 2 \times -3 \times 6$
 $= 18 + 70 + 0 - 84 - 0 - 36$
 $= 40$

4 $\det(A) = (\lambda - 4) \begin{vmatrix} \lambda & 2 \\ 3 & \lambda - 1 \end{vmatrix} = (\lambda - 4)(\lambda^2 - \lambda - 6)$

$\det(A) = 0 \Rightarrow (\lambda - 4)(\lambda^2 - \lambda - 6) = 0$
 $\Rightarrow (\lambda - 4)(\lambda - 3)(\lambda + 2) = 0$
 $\Rightarrow \lambda = 4, 3, -2.$