Questions for review on Math 202

Elements of Differential Equations

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1. State what is meant by Differential Equations.

2. Do you know any application for Differential Equations; give some examples.

3. Write a brief classification with examples of the types of DEs that you studied in your course Math 202.

4. Does every differential equation have a solution.

5. If we know a solution for a given DE, is it necessarily to be unique?

6. What do we mean by an initial value Problem?

7. What do we mean by Cauchy-Euler differential equation? Give an example and show how to solve such type of equations.

8. Complete the following table

Equation	Order	Linear / Nonlinear
$y' = 10 + y^2$		
$x^2\, dy + 5xy\, dx = 0$		
$y = 2xy' + y\,(y')^2$		
$y'' + y = \tan x$		
$y'' - 5y' + 6y = 0$		
$y' + 3x(y'')^3 = \sin x$		
$y' + 3\sin x\, y'' = \cos x$		

9. **Classify** the following 1st Order ODE as **Separable**, **Linear in** ***y*** (or **in** ***x***), **Homogeneous** (**with its degree**), **Bernoulli**, or **Exact**.

i. $(y + y^2)dx - (x + x^2)dy = 0$

ii. $(y - xy^2)dy = ydx$

iii. $(e^{y/x} + e^{x^3/y^3} + 1)dy = (1 + \ln(y/x))dx$

iv. $\dfrac{dy}{dx} = \sqrt{x^2 - y^2}$

v. $3\dfrac{dy}{dx} = 4x - y$

10. Solve $x^2\frac{dy}{dx} = y - xy$

11. Solve $x\frac{dy}{dx} - y = x^2 \sin x$

12. Solve the initial value problem $(e^x + y)dx + (2 + x + ye^y)dy = 0, \ \ y(0) = 1.$

13. Solve the initial value problem $\frac{dy}{dx} = \cos(x + y), \ y(0) = \pi/4$

14. Solve $x\frac{dy}{dx} - (1 + x)y = xy^2$

15. Solve $(y^2 - xy)dx + x^2 dy = 0$

16. Is $y = xe^{-2x}$ a solution to $y'' + 4y' + 4y = 0$?

17. How many solutions are there to the initial value problem $\frac{1}{x^2}\frac{dy}{dx} + y^2 = \frac{1}{x}, \qquad y(0) = 2$. Justify your answer.

18. The Population of a Community is known to increase at a rate Proportional to the number of People present at any time. The Population of the community is doubled after 5 years and it is10,000 after 3 years. What was the initial population. What will be the Population after 10 years.

19. If $y_1 = \ln x$ is a solution of the equation $xy'' + y' = 0$, use **reduction of order** Or an appropriate formula to find a second solution.

20. Solve the boundary value problem: $y'' - 10y' + 25y = 0,, y(0) = 1, \ \ y(1) = 0.$

21. Find the general solution of the following **Cauchy-Euler Equation**

$$2x^2 y'' + 5xy' + y = 0$$

22. Find the solution of the BVP $y^{(4)} + y'' = 0$ satisfying the conditions:

$$y(0) = 0, \ y(\pi) = 0, \ y'(0) = 1, \ y'(\pi) = -1$$

23. Write a homogeneous linear differential equation whose auxiliary equation is

$$5m^5 - 2m^3 + 4m = 0$$

24. Given $y_1 = x \sin(linx)$ a solution of the DE $x^2 y'' - xy' + 2y = 0$. Find another solution for this equation.

25. Using Wronskian show that the functions 1, $1/x$ and $\log x$ are linearly independent on the interval $(0, \infty)$.

26. Show that 1, x, $\sin x$, $\cos x$ form **a Fundamental Set of the solutions** of the Differential Equation $y^{(4)} + y'' = 0$ on $(-\infty, \infty)$.

27. Use the method of **Variation of Parameters** to find the general solution of the differential equation $\frac{d^2y}{dx^2} + y = \sin x$

28. Solve the above question using the method of **Undetermined Coefficients**.

29. Solve the DE: $y''' - xy'' = 8x^2$.

30. If $y_p = u_1y_1 + u_2y_2 + u_3y_3$ is a particular solution of $y^{(3)} + 9y^{(1)} = \tan x$, then find: (i) y_1, y_2, y_3 (ii) u_1', u_2', u_3'

31. Find all Singular Points of the ODE and classify them as regular or irregular singular point: $x^3(x^2-9)y'' - 2x^2(x+3)y' + (x-3)y = 0$

32. Use the **Power Series method** to find the General solution of the DE

$$y'' - 4xy' - 4y = e^x \qquad \text{about } x_0 = 0.$$

33. Show that $x_0 = 0$ is a regular singular point of the differential equation

$$(6x + 2x^3)y'' + 21xy' + 9(x^2-1)y = 0.$$

Then find the **Indicial Equation** and its roots about $x_0 = 0$.

34. Use **Gauss-Jordan Elimination Method**, to solve the system

$$\begin{aligned} s - t + u + v &= 0 \\ 2s + 2u &= 0 \\ s + t + u - v &= 0 \\ -s - 3t - u + 3v &= 0 \end{aligned}$$

35. Find the inverse of A, if it exists, where $A = \begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix}$

36. Find the **eigen values** of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$, and find the corresponding **eigen vectors**.

37. Solve the system

$$\frac{dx}{dt} = x$$
$$\frac{dy}{dt} = 2x + 2y - z$$
$$\frac{dz}{dt} = y$$

38. Solve the system

$$\frac{dx}{dt} = 3x + 4y$$
$$\frac{dy}{dt} = -4x + 3y$$

39. Solve the system

$$X' = \begin{bmatrix} 1 & 3 & -3 \\ 0 & 1 & 0 \\ 6 & 3 & -8 \end{bmatrix} X$$

40. Solve the following non homogeneous system

$$X' = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ te^t \\ e^t \end{bmatrix}$$

41. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix}$.

Compute e^{At} and then use it to find the general solution of the system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix} X .$$