1.1 Definition and Terminology

Notations

Notation	Meaning	Remark	Example
$y = f(x)$	y is a function of x [*Explicit Function*]	i. x is Independent Variable ii. y is Dependent Variable	(1) $y = 3\sin x + 5$ (2) $y = 2\ln x + 3\sqrt{x}$
$f(x, y) = 0$	**Functional Equation** [y as *Implicit Function* of x]	In many cases, y CAN NOT be an Explicit Function of x	(*) $y\cos x + y^2 x = 6$
$\frac{dy}{dx}$ or y'	Ordinary Derivative of y with respect to x	Rate of Change of y with respect to x	(1) $y' = 3\cos x$ (*) $y' = \frac{y\sin x - y^2}{\cos x + 2yx}$
$z = f(x, y)$	z is a function of x, y	i. x, y are Independent Variable ii. z is Dependent Variable	(3) $z = 3\sin(xy) - x$
$\frac{\partial z}{\partial y}$ or z_y	Partial Derivative of z with respect to y	Rate of Change of z with respect to y Variable x will be treated as Constant	(3) $\frac{\partial z}{\partial y} = 3x\cos(xy)$

Differential Equations

An Equation that Contains Derivatives or Partial Derivatives of One or More Dependent Variables with respect to One or More Independent Variables

Classifications of Differential Equations

I. Type of Differential Equations (ODE / PDE)

Ordinary Differential Equation (ODE) contains

Ordinary Derivatives of One or More Dependent Variables w.r.t. a Single Independent Variable

Examples

1. $3\frac{dy}{dx} + 2y = e^{4x}$

2. $\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4x^2 y = 0$

Partial Differential Equation (PDE) contains

Partial Derivatives of One or More Dependent Variables w.r.t. Two or More Independent Variable

Examples

1. $\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$

2. $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial t^2} - 2\frac{\partial u}{\partial t}$

II. Order of Differential Equations

[Order of Highest Derivative in the Equation]

Examples:

1. $3\frac{dy}{dx} + 2y = e^{4x}$ (First Order ODE)

2. $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^4 + 4x^2 y = 0$ (2nd Order ODE)

3. $f(x, y, y', y'', \ldots, y^{(n)}) = 0$ (nth Order ODE)

III. Linear ODE

$$y^{(n)} = f(x, y, y', \ldots, y^{(n-1)})$$

is Linear ODE if f is a Linear Function of $y, y, \ldots, y^{(n-1)}$

	ODE	Order	Linear
1.	$2y' + 3xy = y\sin x$	1	Yes
2.	$2y' + 3xy = \sin y$	1	No
3.	$y''' + 3xy\,y' = \sin x$	3	No
4.	$y' + 3x(y'')^3 = \sin x$	2	No
5.	$y' + 3\sin x\, y'' = \cos x$	2	Yes

Solution of a Differential Equations $f(x, y, y', y'', \ldots y^{(n)}) = 0$

$y = \square(x)$ defined on an Interval **I** is a Solution if

$f(x, \square, \square', \square'', \ldots \square^{(n)}) = 0$

Exercises 1.1

Q.	Equation	Type	Order	Linear / Nonlinear	Solution (Verify!)
15	$y' = 25 + y^2$				$y = 5 \tan x$
19	$x^2\, dy + 2xy\, dx = 0$				$y = -1/(x^2)$
21	$y = 2xy' + y(y')^2$				$y^2 = c_1(x + \frac{1}{4} c_1)$
34	$y'' + y = \tan x$				$y = -\cos x \ln(\sec x + \tan x)$
41	$(y')^2 = 9\, xy$				$y = \begin{cases} 0, & x<0 \\ x^3 & x \geq 0 \end{cases}$
42	$y'' - 5y' + 6y = 0$				$y = e^{mx}$ for some m
43	$x^2 y'' - y = 0$				$y = x^m$ for some m

Type of Solutions of ODE

	Type	Explanation	Examples
1	Explicit Solution	Dependent Variable is Expressed Only in terms of Independent Variable, i.e. $y = \square(x), x \in$ I	Q. 15, 19, 34, 41, 42, 43 have Explicit Solutions on Intervals with some Condition (Can you find it?)
2	Trivial Solution	Solution: $y = 0$ on some interval I [Note: Not Every ODE has a Trivial Sol.]	i. Q. 21, 41-43 : Trivial Solution on R. ii. Q.19 : Trivial Solution on any interval not containing 0. iii.Q. 15, 34 : No Trivial Solution.
3	Implicit Solution	A Solution of the Form : G(x, y) = 0 on an Interval I where at least One Function a) $y = \square(x)$ satisfies the ODE b) $y = \square(x)$ **is an** implicit function of x .	i. Q. 21: The solution is Implicit.

1. **i. <u>Integral of ODE</u>:** Solution of ODE **ii.** <u>Integral Curve</u>: Graph of Solution

2. <u>One-Parameter Family of Solutions</u>**:** A Solution **G(**x,y,c**)=0** of 1st Order ODE **F(** x,y,y' **)=0**
[Note: The solution contains One Parameter c, the constant due to one Integration**]**

3. **<u>n- Parameter Family of Solutions</u>: G(**$x,y,c_1,\ldots c_n$**)=0** for nth Order ODE **F(** $x,y,y',\ldots,y^{(n)}$ **)=0**

4. <u>Particular Solution</u>**:** A Solution of ODE that is free of Arbitrary Parameter.[See Solutions: Q 15,19]

5. **<u>Piecewise-Defined Solution</u>:** A Particular Solution of ODE in the Form of Piecewise-Defined Function which can not be obtained from the Parametric Family of Solutions. [See Solution: Q 41]

6. **<u>Singular Solution</u>:** A Solution of ODE that cannot be obtained from the Parametric Family of Solutions simply by the replacement of the Parameter(s) with constant(s).
Example: i. $y = (x^2/4 + c)^2$ is a one-parameter family of Solutions for the ODE $y' = x\sqrt{y}$.
ii. $y = 0$ is also a solution of the same ODE. But it can not be obtained from $y = (x^2/4 + c)^2$