Math 260 – Quiz # 4d

Name: Solution Sr #: ______________

1. Let V be the set of all vectors (x_1, x_2, x_3) in R^3 such that $x_2 = 0$. Prove or disprove that V is a subspace of R^3.

(i) Let $u, v \in V$, so that $u = (x_1, x_2, x_3)$, $v = (y_1, y_2, y_3)$

and we have $x_2 = 0$, $y_2 = 0$ i.e. $u = (x_1, 0, x_3)$, $v = (y_1, 0, y_3)$

Consider $u + v = (x_1 + y_1, x_2 + y_2, x_3 + y_3) = (x_1 + y_1, 0, x_3 + y_3)$

$\Rightarrow u + v \in V$.

(ii) Let c be any scalar, then $cu = c(x_1, x_2, x_3) = (cx_1, cx_2, cx_3)$

$= (cx_1, 0, cx_3)$, since $x_2 = 0$

$\therefore cu \in V$.

Hence V is a subspace of R^3.

2. Determine whether the vectors u, v and w are linearly independent or not, where $u = (3, -2, 2)$, $v = (3, -1, 4)$, $w = (1, 0, 5)$.

Method 1: Consider $c_1 u + c_2 v + c_3 w = 0 \Rightarrow c_1(3,-2,2) + c_2(3,-1,4) + c_3(1,0,5) = (0,0,0)$

$$\Rightarrow \left.\begin{array}{l} 3c_1 + 3c_2 + c_3 = 0 \\ -2c_1 - c_2 = 0 \\ 2c_1 + 4c_2 + 5c_3 = 0 \end{array}\right\}$$

Solving this system we get $c_1 = c_2 = c_3 = 0$

$\therefore$ u, v, w are linearly indep.

Method 2: Consider the determinant, $\begin{vmatrix} 3 & 3 & 1 \\ -2 & -1 & 0 \\ 2 & 4 & 5 \end{vmatrix} = 9 \neq 0$

$\therefore$ u, v, w are linearly indep.