Math 260 – Quiz # 4c

Name: Solution Sr #: __________

1. Let V be the set of all vectors (x_1, x_2, x_3) in R^3 such that $x_1 = 3x_2$. Prove that V is a subspace of R^3.

Let $u, v \in V$ so that $u = (x_1, x_2, x_3)$, $v = (y_1, y_2, y_3)$

and we have: $x_1 = 3x_2$ and $y_1 = 3y_2$.

$u+v = (x_1+y_1, x_2+y_2, x_3+y_3)$

$x_1+y_1 = 3x_2+3y_2 = 3(x_2+y_2)$

$\therefore$ the vector $u+v$ satisfies the condition for the vectors in V

$\Rightarrow$ $u+v \in V$.

Also, for a scalar c, we have $cu = c(x_1, x_2, x_3) = (cx_1, cx_2, cx_3)$

Now, $cx_1 = c(3x_2) = 3(cx_2)$ showing that $cu \in V$

$\therefore$ V is a subspace of $\mathbb{R}^3$.

2. Determine whether the vectors u, v and w are linearly independent or not, where $u = (1, 0, 3)$, $v = (2, 1, 0)$, $w = (3, 1, 3)$.

Method 1: Consider $c_1u+c_2v+c_3w=0 \Rightarrow c_1(1,0,3)+c_2(2,1,0)+c_3(3,1,3)=(0,0,0)$

$$\Rightarrow \left.\begin{aligned} c_1+2c_2+3c_3 &= 0 \\ c_2+c_3 &= 0 \\ 3c_1 \quad\quad +3c_3 &= 0 \end{aligned}\right\}$$

Solving we get $c_1 = 1, c_2 = 1, c_3 = -1$

Hence $\exists$ non-zero scalars c_1, c_2, c_3 such that $c_1u+c_2v+c_3w=0$

$\therefore$ u, v, w are linearly dep.

Method 2: Consider the determinant: $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 3 & 0 & 3 \end{vmatrix} = 0 \Rightarrow u, v, w$ are linearly dep.

Method 3: Observe that $w = u+v$ i.e. one vector is a linear combination of the others. Hence u, v, w are linearly dep.