Math 260 – Quiz # 4a

Name: Solution Sr #: _____

1. Let V be the set of all positive real numbers with the usual operations of addition and multiplication. Prove or disprove that V is a vector space.

Clearly, if $u, v \in V$ then $u+v \in V$. However, for a negative scalar c, we have $cu \notin V$. For example, take $c=-2$ then $-2u$ is not positive, i.e. $-2u \notin V$.

Hence V is not a vector space.

2. Determine whether the vectors u, v and w are linearly independent or not, where $u = (1, 0, 3)$, $v = (2, 1, 0)$, $w = (3, 2, -1)$.

Consider $c_1u + c_2v + c_3w = 0$

$$c_1(1,0,3) + c_2(2,1,0) + c_3(3,2,-1) = (0,0,0)$$

$$\Rightarrow \begin{cases} c_1 + 2c_2 + 3c_3 = 0 \\ c_2 + 2c_3 = 0 \\ 3c_1 - c_3 = 0 \end{cases}$$ solving this system $\Rightarrow c_1 = c_2 = c_3 = 0$

$\therefore u, v, w$ are linearly indep.

Another method:

Consider the determinant: $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 0 & -1 \end{vmatrix} = 2 \neq 0$

$\therefore u, v, w$ are linearly indep.