Math 260 – Quiz # 2C

Name: Solution Sec. _____ ID: _____ Ser. #_____

Solve the following DE by using two different methods: $(x^2+4)y' + 3xy = x$

Method 1: (Linear)

$$(x^2+4)\frac{dy}{dx} + 3xy = x$$

$$\frac{dy}{dx} + \frac{3x}{x^2+4}y = \frac{x}{x^2+4} \quad \text{(Linear DE)}$$

$$\rho(x) = e^{\int \frac{3x}{x^2+4}dx} = e^{\frac{3}{2}\int \frac{2x}{x^2+4}dx} = e^{\frac{3}{2}\ln(x^2+4)} = (x^2+4)^{\frac{3}{2}}$$

$$\frac{d}{dx}\left[y(x^2+4)^{\frac{3}{2}}\right] = \frac{x}{x^2+4}(x^2+4)^{\frac{3}{2}} = x(x^2+4)^{\frac{1}{2}}$$

$$\Rightarrow y(x^2+4)^{\frac{3}{2}} = \int x(x^2+4)^{\frac{1}{2}}dx = \frac{1}{3}(x^2+4)^{\frac{3}{2}} + C$$

$$\boxed{y = \frac{1}{3} + C(x^2+4)^{-\frac{3}{2}}}$$

Method 2: (Separating the variables)

$$(x^2+4)\frac{dy}{dx} + 3xy = x$$

$$\frac{dy}{dx} = \frac{x}{x^2+4} - \frac{3xy}{x^2+4} = \frac{x}{x^2+4}[1-3y] \quad \text{(Separable)}$$

$$\frac{dy}{1-3y} = \frac{x}{x^2+4}dx$$

$$-\frac{1}{3}\ln|1-3y| = \frac{1}{2}\ln(x^2+4) + \ln|K|$$

$$\frac{1}{2}\ln(x^2+4) + \frac{1}{3}\ln|1-3y| + \ln|K| = 0$$

$$\ln\left|(x^2+4)^{\frac{1}{2}}(1-3y)^{\frac{1}{3}}K\right| = 0$$

$$K\sqrt{x^2+4}\sqrt[3]{1-3y} = 1 \Rightarrow \sqrt[3]{1-3y} = \frac{1}{K(x^2+4)^{\frac{1}{2}}}$$

$$1-3y = \frac{1}{K^3(x^2+4)^{\frac{3}{2}}}$$

$$\therefore \boxed{y = \frac{1}{3} + C(x^2+4)^{-\frac{3}{2}}}$$