Math 260 – Quiz # 7b

Name: Solution Sec.#: ______ Sr #: ______________

Find bases for the eigenspaces of $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. Is A diagonalizable?

$$|A-\lambda I|=0 \Rightarrow \begin{vmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{vmatrix} = -\lambda \begin{vmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{vmatrix} - 2 \begin{vmatrix} 1 & 2-\lambda \\ 1 & 0 \end{vmatrix}$$

$$= -\lambda(2-\lambda)(3-\lambda) - 2(-2+\lambda) \quad = (\lambda-2)[\lambda(3-\lambda)-2]$$

$$= (\lambda-2)(\lambda^2-3\lambda+2) \quad = (\lambda-2)(\lambda-2)(\lambda-1) = 0$$

$\Rightarrow$ $\lambda = 2, 2, 1$ are the eigen values of A. [not distinct ?]

We find the corresponding eigenvectors:

$\underline{\lambda = 2}$: $(A-2I)X = 0 \Rightarrow \begin{bmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Solving this system, we get $x_3 = t$, $x_2 = s$, $x_1 = -t$

$\therefore X = \begin{bmatrix} -t \\ s \\ t \end{bmatrix} = \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix} + \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix} = -t \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

The eigenvectors correspond to $\lambda = 2$ are $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. (They are L. indep.)

$\Rightarrow \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ is a basis for the eigen space corresponding to $\lambda = 2$.

$\underline{\lambda = 1}$: $(A-I)X = 0 \Rightarrow \begin{bmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Solving this system, we get $x_2 = x_3 = t$, $x_1 = -2t$

$\therefore X = \begin{bmatrix} -2t \\ t \\ t \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$.

$\Rightarrow \left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis for the eigen space corresponding to $\lambda = 1$.

A is diagonalizable. (You can easily check that $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ are Linearly indep.)