Math 260 – Quiz # 5b

Name: ______________________________ Sec.#:________ Sr #: _______________

Find a basis and the dimension of the solution space of the system: $(\lambda I_3 - A)X = 0$, for

$$A = \begin{bmatrix} -3 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \text{ and } \lambda = -2.$$

<u>Solution</u>:

$$\lambda I_3 - A = -2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -3 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad (\lambda I_3 - A)X = \begin{bmatrix} 1 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ -2 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{2R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\frac{-1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & \frac{-2}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \quad x_3 = r$$
$$x_2 = \tfrac{2}{3}r$$
$$x_1 = -r$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -r \\ \frac{2}{3}r \\ r \end{bmatrix} = r\begin{bmatrix} -1 \\ \frac{2}{3} \\ 1 \end{bmatrix}$$

Hence, a basis for the solution space is $\{(-3, 2, 3)\}$, and its dimension is 1.