Math 260 – Quiz # 12

Name: Solution Sr #: _____

Solve the system

$$\frac{dx}{dt} = 4x + y$$
$$\frac{dy}{dt} = 4y + z$$
$$\frac{dz}{dt} = 4z$$

$$\Rightarrow A = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

$|A - \lambda I| = 0 \Rightarrow (\lambda - 4)^3 = 0 \Rightarrow \lambda = 4, 4, 4$.

$\underline{\lambda = 4}$

$|A - 4I| = 0 \Rightarrow \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow k_3 = 0, k_2 = 0, k_1 = 1 \Rightarrow K_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

$\therefore \vec{X}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{4t}$.

To get $\vec{X}_2$, we find K_2 using: $(A - \lambda I)K_2 = K_1$

$\Rightarrow \left[\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$. Solving this system, we have $K_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

$\therefore \vec{X}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} t e^{4t} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} e^{4t}$

To find $\vec{X}_3$, we need to find K_3 using: $(A - \lambda I)K_3 = K_2$

$\Rightarrow \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$. Solving this system, we get $K_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\therefore \vec{X}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \frac{t^2}{2} e^{4t} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} t e^{4t} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} e^{4t}$

The general solution for the given system is

$$\vec{X} = c_1 \vec{X}_1 + c_2 \vec{X}_2 + c_3 \vec{X}_3$$

$$= c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{4t} + c_2 \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} t e^{4t} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} e^{4t} \right) + c_3 \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \frac{t^2}{2} e^{4t} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} t e^{4t} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} e^{4t} \right)$$