Math 260 – Quiz # 10

Name: Solution Sr #: ____________

Use the method of **undetermined coefficients** to solve the DE: $y'' - y' - 2y = x^2 e^x$ - - - $(*)$

First, we solve $y'' - y' - 2y = 0$

$$\lambda^2 - \lambda - 2 = 0 \Rightarrow (\lambda - 2)(\lambda + 1) = 0 \Rightarrow \lambda = 2, -1$$

$$y_h = c_1 e^{2x} + c_2 e^{-x}$$

We consider a particular solution of the form

$$y_p = (Ax^2 + Bx + C)e^x \text{ - - - - - - - } (**)$$

$$\Rightarrow y_p' = (Ax^2 + Bx + C)e^x + (2Ax + B)e^x = e^x\left[Ax^2 + (2A+B)x + (B+C)\right]$$

$$y_p'' = e^x\left[2Ax + (2A+B)\right] + e^x\left[Ax^2 + (2A+B)x + (B+C)\right]$$

$$= e^x\left[Ax^2 + (4A+B)x + (2A+2B+C)\right]$$

Substitute in the given DE $(*)$: $y_p'' - y_p' - 2y_p = x^2 e^x$

$$e^x\left[Ax^2 + (4A+B)x + (2A+2B+C)\right] - e^x\left[Ax^2 + (2A+B)x + (B+C)\right] - 2(Ax^2 + Bx + C) = x^2 e^x$$

$$e^x\left[-2Ax^2 + (2A-2B)x + (2A+B-2C)\right] = x^2 e^x$$

Equating coefficients, we get:

$$-2A = 1, \quad 2A - 2B = 0, \quad 2A + B - 2C = 0$$

$$\Rightarrow \boxed{A = -\tfrac{1}{2}}, \quad \boxed{B = A = -\tfrac{1}{2}}, \quad \boxed{C = -\tfrac{3}{4}}$$

Substitute in $(**)$ to get

$$y_p = \left(-\tfrac{1}{2}x^2 - \tfrac{1}{2}x - \tfrac{3}{4}\right)e^x$$

$\Rightarrow$ The general solution of the given DE $(*)$ is $y = y_h + y_p$

i.e. $$y = c_1 e^{2x} + c_2 e^{-x} - \left(\tfrac{1}{2}x^2 + \tfrac{1}{2}x + \tfrac{3}{4}\right)e^x$$