How to solve n^th order linear DE using the method of **undetermined coefficients**

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + ... + a_1 y' + a_0 y = f(x) \quad ....(1)$$

Remember first:

- The associated homogeneous equation of (1) is

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + ... + a_1 y' + a_0 y = 0 \quad ....(2)$$

- *f*(*x*) is a polynomial or exp., sin, cos or sum and product of these functions.
- The general solution of (1) is $y = y_h + y_p$

  where y_h is the solution of (2) and y_p is a particular solution of (1).

Case I [No duplication]

Step1: Find the complimentary solution y_h by solving (2).

Step2: Guess the form of y_p as we explained earlier. Be careful, if any part of your guessing appears in y_h, then go to case II.

Step3: Substitute for y_p and its derivatives in the given DE.

Step4: Find the constants we introduced in step2 and determine y_p.

Step5: Write the general solution of (1) as $y = y_h + y_p$.

Step6: In case of IVP, use the initial conditions to find the unique solution.

Now see our examples done in the class. Also read examples 5, 6 and 7 on pages 337 – 339 form your book.

Case II [The case of duplication]

i.e. when a function in the assumed y_p is also a solution of the associated hom. DE (2)

Step1: Same as in case I.

Step2: We first make our initial guess for y_p as before. So, suppose we have:

$$y_p = y_{p_1} + y_{p_2} + ...y_{p_m}$$

If any y_{p_i} contains terms that duplicate terms in y_h, then multiply y_{p_i} by x^n, where *n* is smallest positive integer that eliminates that duplication.

Step3 – Step6: same as in case I.

Now see our examples done in the class. Also see examples 8, 9 and 10 on pages 341 – 342.