Integration Formulas (Required for MATH 260)

No	Type	Formula	Exercise I	Exercise II
1	**Power Rule**	$\int (ax+b)^m\,dx = \dfrac{(ax+b)^{m+1}}{a(m+1)} + c;\ m \neq -1$	$\int (2-7x)^{11}\,dx$	$\int \dfrac{1}{(-8x+45)^{21}}dx$
2.	**Substitution**	$\int g(f(x))\,f'(x)dx = \int g(u)du$ where $u = f(x)$	$\int \dfrac{(x-2x^3)dx}{6(x^4-x^2+7)^{\frac{7}{3}}}$	$\int (x^2-6x)(x^3-9x^2-17)dx$
3	**Direct Formulas exp & log**	i. $\int \frac{1}{x}dx = \ln\lvert x\rvert + c$ ii. $\int e^x dx = e^x + c$ iii. $\int a^x dx = \dfrac{a^x}{\ln a} + c;\ a > 0$	i. $\int \dfrac{18}{20x-7}dx$ ii. $\int -71e^{4x+9}dx$ iii. $\int 5\,(2.5)^{-4x+3}\,dx$	i. $\int \dfrac{3x^2-7x}{2x^3-7x^2+5}dx$ ii. $\int \left(3-8x^3\right)e^{-2x^4+3x}dx$ iii. $\int \dfrac{5(6x-7)}{7^{3x^2-7x}}\,dx$
4	**Simple Trig. Function**	i. $\int \sin(ax+b)dx = \dfrac{-1}{a}\cos(ax+b)+c$ ii. $\int \cos(ax+b)dx = \dfrac{1}{a}\sin(ax+b)+c$ iii. $\int \sec^2 x dx = \tan x + c$ iv. $\int \csc^2 x dx = -\cot x + c$ v. $\int \tan x \sec x dx = \sec x + c$ vi. $\int \cot x \csc x dx = -\csc x + c$	i. $\int \sin(6x-4)dx$ ii. $\int \cos(8-9x)dx$ iii. $\int \sec^2(3x+7)dx$ iv. $\int \csc^2(2-5x)dx$ v. $\int \tan 4x \sec 4x dx$ vi. $\int \cot \pi x \csc \pi x dx$	i. $\int x^4 \sin(6x^5-4)dx$ ii. $\int \sin^3 5x \cos 5x dx$ iii. $\int \tan^4 8x \sec^2 8x dx$ iv. $\int \cot^3 x \csc^4 x dx$ v. $\int \dfrac{1}{x^2}\tan\dfrac{\pi}{x}\sec\dfrac{\pi}{x}dx$ vi. $\int x^6 \cot x^7 \,\mathrm{cs}\, x^7 dx$
5	**Use of Inverse Trig. Functions** **Also, see 9**	i. $\int \dfrac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$ ii. $\int \dfrac{dx}{1+x^2} = \tan^{-1} x + c$ iii. $\int \dfrac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$	i. $\int \dfrac{dx}{\sqrt{25-9x^2}}$ ii. $\int \dfrac{dx}{100+49x^2}$ iii. $\int \dfrac{dx}{x\sqrt{16x^2-81}}$	i. $\int \dfrac{dx}{\sqrt{18+8x-x^2}}$ ii. $\int \dfrac{dx}{170+70x+49x^2}$ iii. $\int \dfrac{dx}{(x-9)\sqrt{x^2-18x}}$
6	**Integration by Parts**	Rule: $\int u dv = uv - \int v du$ i. $\int xe^x dx = xe^x - e^x + c;$ $\left[u = x, dv = e^x dx \Rightarrow du = dx, v = \int e^x dx\right]$ ii. $\int x \sin x dx = -x\cos x + \sin x + c$ iii. $\int \ln x dx = x\ln x - x + c;$ $\left[u = \ln x, dv = dx\right]$ iv. $\int \cos x e^x dx = \dfrac{1}{2}\left(\cos x e^x + \sin x e^x\right) + c$	i. $\int x^2 \ln x dx$ ii. $\int x^2 \sin x dx$	i. $\int 6xe^{5x-9}dx$ ii. $\int x^7 \cos x^4 dx$

Use of Trig. Identities in Integration

	Trig. Identities	Exercises
7	i. $\cos^2 x = \frac{1+\cos 2x}{2}$ ii. $\sin^2 x = \frac{1-\cos 2x}{2}$ iii. $\sin 2x = 2\sin x \cos x$	i. $\int \sin^2(3x+1)dx$ ii. $\int \sin^4 5x dx$ iii. $\int x\sin^2(7x^2+3)dx$ iv. $\int x\cos^2(3x+1)dx$ v. $\int \cos^3 5x dx$ vi. $\int x\cos(7x^2+3)dx$ vii. $\int \cos^2 3x \sin^2 3x dx$ viii. $\int x^3 \sin^2(x^2+3)dx$

Use of Partial Fractions Decomposition of Rational Functions $N(x)/D(x)$ in Integration

[*Note: If deg (N) ≥ deg (D), first divide N by D. Then find Partial Fractions Decomposition. **]**

Here, in (i-iv), we assume that **deg (**Numerator) < **deg** (Denominator)

8	Type	Method	Exercises
i	**Non-Repeated Linear Factors**	$\frac{p(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$ [Write: $p(x)=A(x-b)+B(x-a)$]	$\frac{x+11}{x^2-2x-15} = \frac{A}{x+3} + \frac{B}{x-5}$; Ans: $A=-1,\ B=2$
ii	**Repeated Linear Factors**	$\frac{p(x)}{(x-a)(x-b)^2} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{(x-b)^2}$	$\frac{x^2+2x+7}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$; Ans: $A=7,\ B=-6, C=10$
iii	**Non-Repeated Non-Linear Factors**	$\frac{p(x)}{(x^2+bx+c)(x^2+dx+g)} = \frac{Ax+B}{x^2+bx+c} + \frac{Cx+D}{x^2+dx+g}$	$\frac{3x+16}{(x-2)(x^2+7)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+7}$; Ans: $A=2,\ B=-2, C=-1$
iv	**Repeated Non-Linear Factors**	$\frac{p(x)}{(x-a)(x^2+bx+c)^2} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c} + \frac{Dx+E}{(x^2+dx+g)^2}$	$\frac{4x^3+5x^2+7x-1}{(x^2+x+1)^2} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2}$; Ans: $A=4,\ B=1, C=2, D=-2$
*** v**	deg (N) ≥ deg (D)	$\frac{N(x)}{D(x)} = \frac{N(x)}{(x-a)(x-b)} = Q(x) + \frac{R(x)}{(x-a)(x-b)}$; where $\frac{R(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$	$\frac{x^3-4x^2-19x-35}{x^2-7x} = Q(x) + \frac{R(x)}{x^2-7x}$; where $\frac{R(x)}{x^2-7x} = \frac{A}{x} + \frac{B}{x-7}$; Ans: $Q(x) = x+3$; $R(x) = 2x-35$; $A=4,\ B=1, C=2, D=-2$
******	**Trig. Identities:** $\cos^2\theta + \sin^2\theta = 1$; $1+\tan^2\theta = \sec^2\theta$; $1+\cot^2\theta = \csc^2\theta$		