1. Use the $\epsilon - \delta$ definition to prove that $\lim_{x\to 2} \frac{1}{x} = \frac{1}{2}$.

2. Find the limit of each of the following:

   (a) $\lim_{x\to 5} \frac{x^2-25}{|x-5|}$

   (b) $\lim_{x\to 1} \frac{x^3-1}{x^2-1}$

   (c) $\lim_{x\to 2} \frac{x^2-x+6}{x-2}$

   (d) $\lim_{x\to 0^-} \left[\frac{1}{x} - \frac{1}{|x|}\right]$

   (e) $\lim_{x\to -\infty} \frac{\sqrt{9x^6-x}}{x^3+1}$

   (f) $\lim_{x\to 0} \frac{\sin^3 x}{x^3}$

   (g) $\lim_{x\to 0^+} (1+x)^{1/x}$

   (h) $\lim_{x\to 1} \sin^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)$

   (i) $\lim_{x\to \infty} \frac{e^{3x}}{x^4}$

   (j) $\lim_{x\to 0} \frac{\tan x - x}{x^3}$

   (k) $\lim_{x\to 0^+} (\cos x)^{1/x^2}$

3. Find the derivative y' for each of the following:

   (a) $y = \sqrt{e^{2x} - \csc^3 x}$

   (b) $y = \frac{\sec^2 5x^2 + 1}{1 + \cos^{-1} x}$

   (c) $xy = \cot(xy)$

   (d) $yx + 1 = 3\tan^{-1} y$

   (e) $y = \sin(\tan\sqrt{\sin x})$

   (f) $y = 10^{\sin x^2}$

4. If $x^4 + y^4 = 16$, show that $y'' = -48\dfrac{x^2}{y^7}$.

5. Suppose f is a one-to-one differentiable function and its inverse f^{-1} also differentiable. Use implicit differentiation to show that

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'[f^{-1}(x)]}\,, \quad \text{where} \quad f' \neq 0.$$

6. Use the definition of derivative to show that $\dfrac{d}{dx}[\log_b x] = \dfrac{1}{x \ln b}\,, \quad x > 0.$

7. show that the equation $4x^3 - 6x^2 + 3x - 2 = 0$ has a real root between 1 and 2.

8. Show that the function $f(x) = |x - 3|$ is continuous everywhere.

9. Given

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ e^x & \text{if } x < 0. \end{cases}$$

Discuss the continuity of f at $x = 0$.

10. Find the horizontal and vertical asymptotes of the graph of $f(x) = \dfrac{\sqrt{2x^2+1}}{3x-5}$.

11. Find the critical points of $f(x) = 4x^{3/5} - x^{8/5}$.

12. Find the absolute max and absolute min of $f(x) = x^4 - 2x^2 + 3$ on $[-2, 3]$.

13. Sketch the graph of $\dfrac{2x-5}{x+3}$.

14. State Rolle's theorem and verify that the function $f(x) = \sin 2\pi x$ satisfies the hypotheses of Rolle's theorem on the interval $[-1, 1]$. Then find a numnber c that satisfies its conclusion on this interval.

15. Is it true that the equation $y = y''' + 5y' - 6$ is satisfied by $y = x$?

16. Is it true that the inverse function of $y = \sin x$ is $y = \dfrac{1}{\sin x}$?

17. Is it true that the function $y = \ln x$ is differentiale everywhere?

18. Is it true that if $k(x) = f(g(x))$, then $\dfrac{d^2k}{dx^2} = f'(g) \cdot g'' + f''(g) \cdot (g')^2$?

19. What is the error in the following steps:

$$\lim_{x \to 0} \frac{\sin x}{x^2} = \lim_{x \to 0} \frac{\cos x}{2x} = \lim_{x \to 0} \frac{-\sin x}{2} = 0$$

and determine the correct value of this limit.

20. Use local linear approximation to approximate $\sin 29°$.