$\boxed{\text{Ex \# 30} \over 153}$ (a) $f(x) = \dfrac{x^2-4}{x^3-8}$

$f$ is not continuous at $x=2$ since it is not defined at this point.

$$\lim_{x\to 2} \frac{x^2-4}{x^3-8} = \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)} = \lim_{x\to 2} \frac{x+2}{x^2+2x+4} = \frac{1}{3}$$

So, it is a removable discontinuity at $x=2$.

(b) Note that $\lim_{x\to 2} f(x)$ does not exist. So $f$ is not continuous at $x=2$.

It is not removable discontinuity.

(c) $f(1) = 6$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} (3x^2+5) = 8 \neq f(1)$$

$\Rightarrow$ $f$ is not continuous at $x=1$.

$f$ has a removable discontinuity at $x=1$

$\boxed{\dfrac{\text{Ex \# 41}}{154}}$

Volume of a right cylinder of height $h$ and radius $r$ is $\pi r^2 h$

Volume of a cone of height $h$ and radius $r$ is $\frac{1}{3}\pi r^2 h$

Now, both have the same hight $h$, so fix $h$.

Consider the function

$$f(r) = \pi r^2 h \,,$$

which gives the volume of a right circular cylinder of height $h$ ad radius $r$.

Note that $\quad 0 < \frac{1}{3}\pi r^2 h < \pi r^2 h$

i.e. $\quad 0 < \frac{1}{3}\pi r^2 h < f(r)$

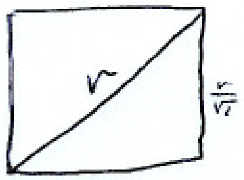
By the Intermediate value Theorem, $\exists$ a number $c$ between $0$ and $r$ such that $f(c) = \frac{1}{3}\pi r^2 h$, so the cylinder of radius $c$ and height $h$ has volume equal to that of the cone of the same height $h$.

$\boxed{\begin{array}{c}\text{Ex \# 42} \\ \hline 154\end{array}}$

Area of square whose diagonal is of length $r$ is $\frac{r^2}{2}$.

i.e. $A = f(r) = \frac{r^2}{2}$

$f(2r) = 2r^2$




$\frac{1}{2}$ area of a circle of radius $r$ $= \frac{1}{2}\pi r^2$

Note that $\quad \frac{r^2}{2} < \frac{1}{2}\pi r^2 < 2r^2$

i.e. $\quad f(r) < \frac{1}{2}\pi r^2 < f(2r)$

By the intermediate value Theorem, $\exists$ a number $c$ between $r$ and $2r$

such that $\quad f(c) = \frac{1}{2}\pi r^2$,

that is a square of diagonal $c$ with area $\frac{1}{2}\pi r^2 = \frac{1}{2}$ area of a circle of radius $r$.