Math 101 Quiz # 2b

Name: Solution I.D. # ______ Section # ____ Serial #

1. Find each of the following limits:

i) $\lim\limits_{x\to 0}\dfrac{\tan 4x}{\sin 5x}$

$$=\lim_{x\to 0}\frac{\frac{\tan 4x}{x}}{\frac{\sin 5x}{x}}=\lim_{x\to 0}\frac{4\,\frac{\tan 4x}{4x}}{5\,\frac{\sin 5x}{5x}}=\frac{4}{5}\lim_{x\to 0}\frac{\frac{\tan 4x}{4x}}{\frac{\sin 5x}{5x}}=\frac{4}{5}$$

ii) $\lim\limits_{\theta\to 0}\dfrac{9}{\theta\csc\theta}$

$$=\lim_{\theta\to 0}\frac{9}{\frac{\theta}{\sin\theta}}=\lim_{\theta\to 0}\frac{9\sin\theta}{\theta}=9\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=9(1)=9$$

iii) $\lim\limits_{x\to 2}\dfrac{\sin \pi x}{x-2}$ (Hint: use the substitution $x-2=t$)

Put $x-2=t$ then $x=t+2$ and $t\to 0$ as $x\to 2$.

$$\lim_{x\to 2}\frac{\sin \pi x}{x-2}=\lim_{t\to 0}\frac{\sin\pi(t+2)}{t}=\lim_{t\to 0}\frac{\sin(\pi t+2t)}{t}=\lim_{t\to 0}\frac{\sin\pi t}{t}$$

$$=\pi\lim_{t\to 0}\frac{\sin\pi t}{\pi t}=\pi(1)=\pi.$$

2. Let $f(x)=\dfrac{1}{x+1}$

i) Find the average rate of change of y with respect to x over the interval [1, 3].

$$r_{ave}=\frac{f(3)-f(1)}{3-1}=\frac{\frac{1}{3+1}-\frac{1}{1+1}}{2}=\frac{\frac{1}{4}-\frac{1}{2}}{2}=-\frac{1}{8}$$

ii) Find the instantaneous rate of change of y with respect to x at $x=1$.

$$r_{inst}=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{\frac{1}{1+h+1}-\frac{1}{1+1}}{h}=\lim_{h\to 0}\frac{\frac{1}{h+2}-\frac{1}{2}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{\cancel{2}-h-\cancel{2}}{2(h+2)}}{h}=\lim_{h\to 0}\frac{-\cancel{h}}{2\cancel{h}(h+2)}=\lim_{h\to 0}\frac{-1}{2(h+2)}=\frac{-1}{4}$$