***Department of Mathematical Sciences, KFUPM**

Time: 50 Min. **MATH 101-051** **Points:_____/50**

ID_________ **Section: 4, 8, 12, 17** **Serial #:_____**

Note: Show all work. Solution **No Calculator is allowed.**

Evaluate the limits in Questions 1-5

1. $\lim_{t\to 0}\frac{\sqrt{t+7}-\sqrt{7}}{t}=$ (6 pts)

$$= \lim_{t\to 0} \frac{\sqrt{t+7}-\sqrt{7}}{t} \cdot \frac{\sqrt{t+7}+\sqrt{7}}{\sqrt{t+7}+\sqrt{7}}$$

$$= \lim_{t\to 0} \frac{t+7-7}{t\left(\sqrt{t+7}+\sqrt{7}\right)} = \lim_{t\to 0} \frac{\cancel{t}}{\cancel{t}\left(\sqrt{t+7}+\sqrt{7}\right)}$$

$$= \lim_{t\to 0} \frac{1}{\sqrt{t+7}+\sqrt{7}}$$

$$= \frac{1}{2\sqrt{7}}$$

2. $\lim_{x\to\infty}\frac{\sqrt[4]{16x^8-809x+10}}{6x^2-1}$ (6 pts)

$$= \lim_{x\to\infty} \frac{\sqrt[4]{\frac{16x^8}{x^8}-\frac{809x}{x^8}+\frac{10}{x^8}}}{\frac{6x^2}{x^2}-\frac{1}{x^2}} \qquad \left[\sqrt[4]{x^8}=x^2\right]$$

$$= \lim_{x\to\infty} \frac{\sqrt[4]{16-\frac{809}{x^7}+\frac{10}{x^8}}}{6-\frac{1}{x^2}} = \frac{2}{6}$$

$$= \frac{1}{3}$$

3. $\lim_{x\to 3^-} \frac{3x-9}{|3-x|} =$ (5 pts)

$$= \lim_{x\to 3^-} \frac{3(x-3)}{|3-x|}$$

$$= \lim_{x\to 3^-} \frac{3(x-3)}{3-x} = \lim_{x\to 3^-} \frac{-3\cancel{(3-x)}}{\cancel{3-x}}$$

$$= -3$$

4. $\lim_{x\to -\infty} \left(300 - 500x^{35} - 20x^{37} - 70x^2\right)$ (4 pts)

$$= \lim_{x\to -\infty} \left(-20x^{37}\right) = +\infty$$

5. $\lim_{t\to 1} \frac{t^2-4t+3}{t^3+t-2}$ (6 pts)

Note that as $t\to 1$, the given limit is of the form $\frac{0}{0}$. So $(t-1)$ is a factor in both the numerator and the denominator. For the numerator, in this case, we may use synthetic division to get a factorization for t^3+t-2 as follows:

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

$$t^3+t-2 = (t-1)(t^2+t+2)$$

$$= \lim_{t\to 1} \frac{\cancel{(t-1)}(t-3)}{\cancel{(t-1)}(t^2+t+2)}$$

$$= \lim_{t\to 1} \frac{t-3}{t^2+t+2}$$

$$= \frac{-2}{4}$$

$$= -\frac{1}{2}$$

6. Let $f(t)=\dfrac{1-t}{\sqrt{t^2-1}}$. Using the **concept of limit**, find (6 pts)

(a) all **horizontal asymptotes** (if any)

$$\lim_{t\to+\infty}\frac{1-t}{\sqrt{t^2-1}}=\lim_{t\to+\infty}\frac{\frac{1}{|t|}-\frac{t}{|t|}}{\sqrt{\frac{t^2}{t^2}-\frac{1}{t^2}}}$$

$$=\lim_{t\to+\infty}\frac{\frac{1}{t}-\frac{t}{t}}{\sqrt{1-\frac{1}{t^2}}}=\lim_{t\to+\infty}\frac{\frac{1}{t}-1}{\sqrt{1-\frac{1}{t^2}}}=-1$$

Also,

$$\lim_{t\to-\infty}\frac{1-t}{\sqrt{t^2-1}}=\lim_{t\to-\infty}\frac{\frac{1}{|t|}-\frac{t}{|t|}}{\sqrt{\frac{t^2}{t^2}-\frac{1}{t^2}}}=\lim_{t\to-\infty}\frac{-\frac{1}{t}+1}{\sqrt{1-\frac{1}{t^2}}}=1$$

Hence, $y=1$ and $y=-1$ are horizontal asymptotes.

(b) all **vertical asymptotes** (if any)

[Note that the denominator is zero when $t=1, t=-1$]

$$\lim_{t\to1}\frac{1-t}{\sqrt{t^2-1}}=\lim_{t\to1}\frac{1-t}{\sqrt{t^2-1}}\cdot\frac{\sqrt{t^2-1}}{\sqrt{t^2-1}}$$

$$=\lim_{t\to1}\frac{(1-t)\sqrt{t^2-1}}{t^2-1}$$

$$=\lim_{t\to1}\frac{-\cancel{(t-1)}\sqrt{t^2-1}}{\cancel{(t-1)}(t+1)}$$

$$=\lim_{t\to1}\frac{-\sqrt{t^2-1}}{t+1}=\frac{0}{2}=0\Rightarrow \text{No V.A. at } t=1.$$

$\lim_{t\to-1}\frac{1-t}{\sqrt{t^2-1}}$ DNE, since $\lim_{t\to-1}(1-t)=2$ but $\lim_{t\to-1}\sqrt{t^2-1}=0$

$\therefore\ t=-1$ is a vertical asymptote.

7. **Use the (ε,δ) definition of limit** to show that $\lim_{x\to -1}(3x+7)=4$. (5 pts)

We need to show that for any $\epsilon>0$, $\exists\, \delta>0$ such that

$$|(3x+7)-4|<\epsilon \quad \text{if} \quad 0<|x-(-1)|<\delta$$

i.e. $\quad |3x+3|<\epsilon \quad$ if $\quad 0<|x+1|<\delta$.

Now,

$$|(3x+7)-4| = |3x+3| = 3|x+1| < 3\delta \quad (\text{when } |x+1|<\delta)$$

letting $\epsilon=3\delta$, so that $\delta=\epsilon/3$, we have

for $\epsilon>0$, $\exists\,\delta>0$ such that $|(3x+7)-4|<\epsilon$ if $0<|x+1|<\delta$.

This shows that $\lim_{x\to -1}(3x+7)=4$.

8. **Find all the points** where the function $f(x)=\begin{cases} 8 & \text{if} \quad x=1 \\ \dfrac{4x^2-4}{x-1} & \text{if} \quad 1<x<3 \\ 12 & \text{if} \quad x=3. \end{cases}$

is **not continuous** in the interval $[1,3]$. (6 pts)

We need to check the continuity of f on $(1,3)$ and on the right of 1 and on the left of 3, and also we check the changing point since f is a piecewise function. Note that f is continuous when it is a polynomial.

<u>Now,</u>

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}\frac{4x^2-4}{x-1} = \lim_{x\to 1^+}\frac{4(x^2-1)}{x-1} = \lim_{x\to 1^+}\frac{4\cancel{(x-1)}(x+1)}{\cancel{x-1}} = 8 = f(1)$$

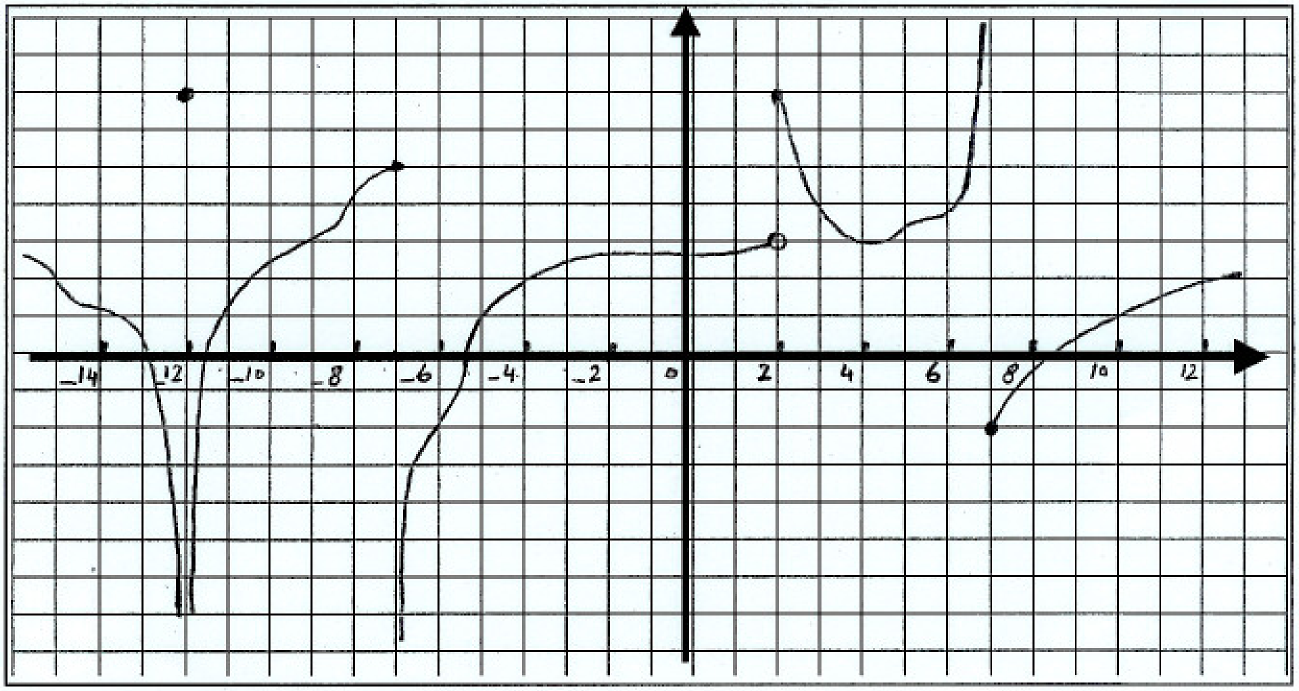
$\Rightarrow$ f is continuous at the right of $x=1$.

<u>Next</u>

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}\frac{4x^2-4}{x-1} = 16 \neq 12 \quad \text{i.e.} \quad \lim_{x\to 3^-} f(x) \neq f(3)$$

$\therefore$ the given function has discontinuity at $x=3$.

9. **Looking at the given graph** of $f(x)$, **answer** the following questions: (6 pts)



a. $f(x)$ is **Not Left Continuous** at the point(s) $x =$ $-12, 2, 7$

b. $f(x)$ is **Not Right Continuous** at the point(s) $x =$ $-12, -7$

c. $\lim_{x \to 2^-} f(x) =$ 3

d. $\lim_{x \to 7^+} f(x) =$ -2

e. $\lim_{x \to -12} f(x) =$ $-\infty$

f. $f(x)$ is **Not Continuous** at the point(s) $x =$ $-12, -7, 2, 7$