

How to Use Mathematica to solve Some Problems of Mathematics

1. Introduction

With the help of Mathematica, we can do

- Arithmetic and Algebraic Manipulations,
- Solve Different Types of Equations,
- Sketch Graphs of Functions and Equations.

2. How to Start

- Click on the icon “Mathematica”.
- Click on the new file.
- Type your operation
- Press the keys:

Shift

 &

Enter

(We shall call these two keys: [SE] keys)

- You will find the Answer in the Next Line.

3. Basic Algebra Operations

Addition: + Subtraction: - Square root: **sqrt**
 Division: / Power: ^ Multiplication: *

(Example)

Example 1. Type: 5+3 **outside the Existing Cell**

- Press the [SE] keys.
- You will get on the screen:

$$\begin{aligned} \text{In [1]} &:= 5+3 \\ \text{Out [1]} &= 8 \end{aligned}$$

Example 2. Type: -7^3 **outside the Existing Cell**

- Press the [SE] keys.
- You will get on the screen:

$$\begin{aligned} \text{In [2]} &:= -7^3 \\ \text{Out [2]} &= -343 \end{aligned}$$

Example 3. Type: $4 + \frac{1}{4}$ **outside the Existing Cell**

- Press the [SE] keys.
- You will get on the screen:

$$\begin{aligned} \text{In [3]} &:= 4 + \frac{1}{4} \\ \text{Out [3]} &= \frac{17}{4} \end{aligned}$$

Example 4. Type: $4. + \frac{1}{4}$

- Press the [SE] keys.
- You will get on the screen:

$$\begin{aligned} \text{In [4]} &:= 4. + \frac{1}{4} \\ \text{Out [4]} &= 4.25 \end{aligned}$$

[Check the difference between Examples 3 & 4]

Example 5. Type: $2^{(1/3)}$

- Press the [SE] keys.
- You will get on the screen:

$$\begin{aligned} \text{In [5]} &:= 2^{(1/3)} \\ \text{Out [5]} &= 2^{1/3} \end{aligned}$$

Example 6. Type: $N[2^{(1/3)}$

- Press the [SE] keys.
- You will get on the screen:

$$\begin{aligned} \text{In [6]} &:= N[2^{(1/3)}] \\ \text{Out [6]} &= 1.25992 \end{aligned}$$

Example 7. Type: $2^{(1/3)}/N$

- Press the [SE] keys.
- You will get on the screen:

$$\begin{aligned} \text{In [7]} &:= 2^{(1/3)}/N \\ \text{Out [7]} &= 1.25992 \end{aligned}$$

[Check the difference among Examples 5, 6 & 7]

4. Built in Functions

For	Type	For	Type
$ -2 $	Abs[-2]	e^2	Exp[2]
sin(5 radian)	Sin[5]	$\sin^{-1}(.3)$	ArcSin[.3]
sin(5 Degrees)	Sin[5 Degree]	sinh(5)	Sinh[5]
Natural Log of 5	Log[5]	Log of 5 to base 3	Log[3,5]

Try the following Exercises

- Find $\cos(30)$ and $\cos(30^\circ)$. Use the methods of Examples 5 & 7. Explain the answers.
- Find $\ln(7.8)$, $\log(15)$, $\log_4(9)$, e^8 , $\tan^2(12)$
- Evaluate: $5-2(8^2-60)/4$ [Ans: 3]
- Evaluate: $\sqrt{9} - \frac{1}{3} + \frac{4(18-4^3)}{8}$
- Evaluate: $2\sin^{-1}(1/3) - \cosh^2(4/5)$.

5. Manipulating Algebraic Expressions

Example 1: Find the value of $6-3x^5$ for $x=3$.

Solution: Type: $6-3*x^5/.x \rightarrow 3$

- Press the [SE] keys. You will get:

$$\begin{aligned} \text{In [6]} &:= 6-3*x^5/.x \rightarrow 3 \\ \text{Out [6]} &= -723 \end{aligned}$$

Example 2: Find the value of $\sqrt{x^2+y^2}$ when $y=x+1$ and $x=3$.

Solution: Type: $x=3; y=x+1; \text{sqrt}[x^2+y^2]$

Press the [SE] keys. You will get:

$$\begin{aligned} \text{In [6]} &:= x=3; y=x+1; \text{sqrt}[x^2+y^2] \\ \text{Out [6]} &= 5 \end{aligned}$$

Example 3: Expand $(x+y)^5$.

Solution: Type: **Clear[x,y]; Expand[(x+y)^5]**

Press the [SE] keys. You get

$$\begin{aligned} \text{In [6]} &:= \text{Clear}[x,y]; \text{Expand}[(x+y)^5] \\ \text{Out [6]} &= x^5 + 5xy^4 + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5 \end{aligned}$$

6. Solving Equations Numerically

Example 1: (Eq. of one variable)

Solve the equation $x^2 + x = 2$ in x .

Solution: Type: `Solve[x^2+x==2, x]`

Press the [SE] keys. You will get:

In [1]: `Solve[x^2+x==2, x]`

Out [1]: $\{\{x \rightarrow -2\}, \{x \rightarrow 1\}\}$

Example 2: (Eq. of two variables)

Solve the equation $x^2 - 4 = 0, y^2 = x^2$ in x & y .

Solution: Type:

`Solve[{x^2-4==0, x^2=y^2}, {x,y}]`

Press the [SE] keys. You will get:

In [2]: =

`Solve[{x^2-4==0, x^2=y^2}, {x,y}]`

Out [2]: =

$\{\{x \rightarrow -2, y \rightarrow -2\}, \{x \rightarrow -2, y \rightarrow 2\}, \{x \rightarrow 2, y \rightarrow -2\}, \{x \rightarrow 2, y \rightarrow 2\}\}$.

Example 3: (Complicated Equation)

Solve the equation $\ln(x + \sqrt{1+x^2}) = 2$ in x .

Solution: Type:

`Solve[Log[x+Sqrt[1+x^2]]==2, x]`

Press the [SE] keys. You will get:

In [3]: =

`Solve[Log[x+Sqrt[1+x^2]]==2, x]`

Out [3]: $\left\{\left\{x \rightarrow \frac{1}{2}e^{-2}(-1+e^4)\right\}\right\}$.

7. Solving Equations Symbolically

Example 1: (Eq. of two variables)

Solve the equation $x^2 - k^2 = 0, y^2 = x^2$ in x & y .

Solution: Type:

`Solve[{x^2-k^2==0, x^2=y^2}, {x,y}]`

Press the [SE] keys. You will get:

In [2]: =

`Solve[{x^2-k^2==0, x^2=y^2}, {x,y}]`

Out [2]: =

$\{\{x \rightarrow -k, y \rightarrow -k\}, \{x \rightarrow -k, y \rightarrow k\}, \{x \rightarrow k, y \rightarrow -k\}, \{x \rightarrow k, y \rightarrow k\}\}$.

Example 2: (Complicated Equation)

Solve the equation $\ln(x + \sqrt{a+x^2}) = b$ in x .

Solution: Type:

`Solve[Log[x+Sqrt[a+x^2]]==b, x]`

Press the [SE] keys. You will get:

In [3]: =

`Solve[Log[x+Sqrt[a+x^2]]==b, x]`

Out [3]: $\left\{\left\{x \rightarrow \frac{1}{2}e^{-b}(-a+e^{2b})\right\}\right\}$.

8. Numerical Solutions of Equation(s)

Example 1: Find the roots of the equation:

$$x^3 + x + 1 = 0.$$

Solution: Type:

`NSolve[x^3+x+1==0, x]`

Press the [SE] keys. You will get:

In [1]: `NSolve[x^3+x+1==0, x]`

Out [2]: =

$\left\{\left\{x \rightarrow -0.682328\right\}, \left\{x \rightarrow 0.341164 - 1.16154i\right\}, \left\{x \rightarrow 0.341164 + 1.16154i\right\}\right\}$.

Example 2: Find solution of the system of equations:

$$x + y = 2, x - 3y + z = 3, x - y + z = 0.$$

Solution: Type:

`NSolve[{x+y==2, x-3y+z==3, x-y+z==0}, {x,y,z}]`

Press the [SE] keys. You will get:

In [2]: =

`NSolve[{x+y==2, x-3y+z==3, x-y+z==0}, {x,y,z}]`

Out [2]: $\{\{x \rightarrow 3.5, y \rightarrow -1.5, z \rightarrow -5\}\}$.

Example 3: Find approximate solution of the equation:

$$3 \cos x = \ln x$$

starting the approximation at $x=1$

Solution: Type:

`FindRoots[3 cos[x] == Log[x], {x, 1}]`

Press the [SE] keys. You will get:

In [3]: `FindRoots[3 cos[x] == Log[x], {x, 1}]`

Out [3]: $\{x \rightarrow 1.44726\}$.

9. Sketching 2-D Graphs

Example 1: Draw graph of $y = \sin x$ when $0 \leq x \leq \pi$.

Solution: Type `Plot[cos[x], {x, 0, pi}]`

Press the [SE] keys. You will get:

In [3]: `Plot[cos[x], {x, 0, pi}]`

Out [3]: **You find the graph of $\cos x$ when $0 \leq x \leq \pi$.**

Example 2: Draw graph of $f(x) = \begin{cases} x^3 - 1, & x \geq 0 \\ x^2, & x < 0. \end{cases}$

when $-4 \leq x \leq 5$.

Solution: Type

`f[x_]:=x^3-1 /; x>=0`

`f[x_]:=x^2 /; x<0.`

Press the [SE] keys. You will get:

In [3]: `f[x_]:=x^3-1 /; x>=0`

`f[x_]:=x^2 /; x<0.`

Out [3]: