Section # __________ **Quiz 5 (MATH 102)** Sr # __________

**Q1**. Find the Maclaurin's Polynomial of ln(2x+1).

(1) $f(x) = \ln(2x+1)$ $\quad f(0) = \ln 1 = 0$

$f'(x) = (2x+1)^{-1}$

$f''(x) = (-1)(2x+1)^{-2}(2)$

$f'''(x) = (-1)(-2)(2x+1)^{-3}(2)^2$

$\vdots$

$f^{(k)}(x) = (-1)(-2)\cdots(-(k-1))(2x+1)^{-k}\,2^{k-1}$

$= (-1)^{k-1}(k-1)!\,2^{k-1}(2x+1)^{-k}$

$\Rightarrow \boxed{f^{(k)}(0) = (-1)^{k-1}(k-1)!\,2^{k-1} \quad k \geq 1}$

(2) $p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k$

$= \sum_{k=1}^{n} \frac{(-1)^{k-1} 2^{k-1}}{k} x^k$

**Q2.** If we want to approximate ln(sin 91$^0$) by using Taylor Series of a function $f(x)$ about $x = a$, then (fill in the blanks)

$f(x) =$ $\ln(\sin x)$ and $a =$ $\pi/2$ $(=90^\circ)$

**Q3**. Evaluate: $\lim_{n\to\infty} \frac{\ln(5n^2+1)}{3n^2}$.

$f(x) = \ln(5x^2+1)/3x^2$ $\quad (\infty/\infty)$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \left(\frac{1}{5x^2+1}\cdot 10x\right)/6x$

$= \lim_{x\to\infty} \frac{10x}{6x(5x^2+1)} = \lim_{x\to\infty} \frac{10}{6(5x^2+1)} = 0$

$\Rightarrow \boxed{\lim_{n\to\infty} \frac{\ln(5n^2+1)}{3n^2} = 0}$

**Q4.** Write the general term of the sequence $\left\{\sum_{k=1}^{n} 5(3)^{4-k}\right\}_{n=1}^{\infty}$ in closed form and then evaluate its limit.

$a_n = \sum_{k=1}^{n} 5(3)^{4-k} = 5\cdot 3^4 \sum_{k=1}^{n} 3^{-k}$

for $\sum_{k=1}^{n} 3^{-k} = \frac{a(1-r^n)}{1-r} = \frac{1}{3}\left(\frac{1-3^{-n}}{1-3^{-1}}\right)$

$\Rightarrow \boxed{a_n = \frac{5\cdot 3^4}{3}\left(\frac{1-3^{-n}}{1-3^{-1}}\right)}$ Closed form

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{5\cdot 3^4}{3}\left(\frac{1-3^{-n}}{1-3^{-1}}\right)$

$= \frac{5\cdot 3^4}{3}\left(\frac{1}{1-3^{-1}}\right) = \frac{5\cdot 3^4}{2}$

$\boxed{\text{Note: Sum of } \sum_{k=1}^{\infty} 5(3)^{4-k} \text{ is } \frac{5\cdot 3^4}{2}}$

**Q5**. Check if the sequence $\left\{\frac{5^n}{2^{(n^2)}}\right\}_{n=1}^{\infty}$ is strictly increasing or decreasing.

Here $a_n = \frac{5^n}{2^{(n^2)}}$

$a_{n+1} = \frac{5^{n+1}}{2^{(n+1)^2}}$

$\Rightarrow \frac{a_{n+1}}{a_n} = \frac{5^{n+1}}{2^{(n+1)^2}} \div \frac{5^n}{2^{(n^2)}} = \frac{5}{2^{2n+1}} < 1$ for $n \geq 3$

$\Rightarrow$ The sequence $\left\{\frac{5^n}{2^{(n^2)}}\right\}_{n=1}^{\infty}$ is Eventually Decreasing