Name Solution **MATH 102 - Quiz 3** I.D. #

Determine about the convergence or divergence of each of the following series:

(1) $\sum_{k=1}^{\infty}\left(\frac{5k}{3k+2}\right)^k$

Using the root test, we have

$$\lim_{k\to\infty} (a_k)^{\frac{1}{k}} = \lim_{k\to\infty}\left[\left(\frac{5k}{3k+2}\right)^k\right]^{\frac{1}{k}}$$

$$= \frac{5}{3} > 1$$

$\therefore$ the series diverges.

(2) $\sum_{k=1}^{\infty}\frac{e^k}{k!}$

By the ratio test,

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{e^{k+1}}{(k+1)!}\cdot\frac{k!}{e^k}$$

$$= \lim_{k\to\infty} \frac{e}{k+1}$$

$$= 0 < 1$$

$\therefore$ the series converges

(3) $\sum_{k=1}^{\infty}\frac{1+|\sin k|}{k^4}$

We use Comparison test:

first, note that $|\sin k| \le 1$

$$\frac{1+|\sin k|}{k^4} \le \frac{2}{k^4}$$

So, we consider $\sum_{k=1}^{\infty}\frac{2}{k^4}$ which is

convergent [p-series],

and $\sum_{k=1}^{\infty}\frac{1+|\sin k|}{k^4} \le \sum_{k=1}^{\infty}\frac{2}{k^4}$

$\therefore$ by the comparison test, the series $\sum_{k=1}^{\infty}\frac{1+|\sin k|}{k^4}$ converges.

(4) $\sum_{k=0}^{\infty}\frac{k^k}{7^{k+1}}$

Using the root test, we have

$$\lim_{k\to\infty} (a_k)^{\frac{1}{k}} = \lim_{k\to\infty}\left(\frac{k^k}{7^{k+1}}\right)^{\frac{1}{k}}$$

$$= \lim_{k\to\infty}\left[\left(\frac{k}{7}\right)^k\right]^{\frac{1}{k}}\cdot\left(\frac{1}{7}\right)^{\frac{1}{k}}$$

$$= \lim_{k\to\infty}\left(\frac{k}{7}\right)\cdot\left(\frac{1}{7}\right)^{\frac{1}{k}}$$

$$= \infty$$

$\therefore$ the series diverges.