King Fahd University of Petroleum and Minerals
Math 102
Practice Exam Covering Ch. 7 & 8.2

Name: Solution Sec: ______ ID: ____________

For questions 1 – 7, circle the right answer:

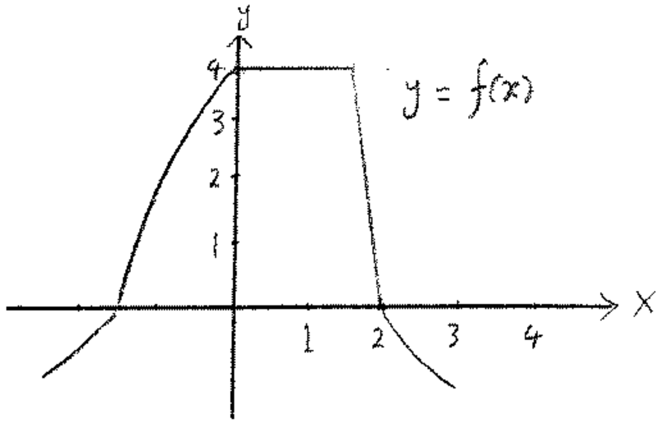
1. The value of k which gives the largest value of the integral $\int_0^k f(x)dx$ is:

(a) 1

(b) 2

(c) 3

(d) 4



2. The area of the region enclosed by $x = 3y - y^2$ and $x + y = 3$ is

(a) $\frac{4}{3}$

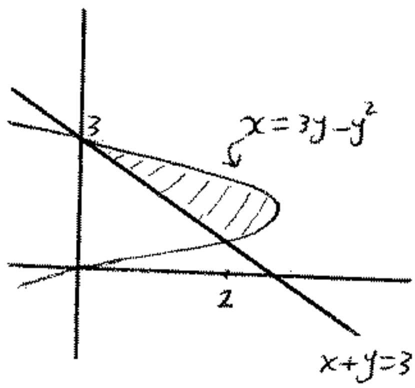
(b) $\frac{3}{4}$

(c) 3

(d) 4

To find pts of intersections, we solve:

$$\left.\begin{aligned} x &= 3y - y^2 \\ x &= 3 - y \end{aligned}\right\} \Rightarrow \begin{aligned} &y^2 - 4y + 3 = 0 \\ &(y-1)(y-3) = 0 \\ &y = 1,\ y = 3 \end{aligned}$$



$$A = \int_1^3 \left[(3y - y^2) - (3 - y)\right] dy$$

$$= \int_1^3 (-y^2 + 4y - 3)\, dy$$

$$= \left[-\frac{y^3}{3} + 2y^2 - 3y\right]_1^3 = \frac{4}{3}$$

3. Using cylindrical shells, the volume of the solid that results when the region enclosed by $y = x^2$, $y = 0$ and $x = 3$, is revolved about the line $x = 3$:

(a) $\frac{27}{2}\pi$

(b) $\frac{5}{3}\pi$

(c) 3π

(d) 5π

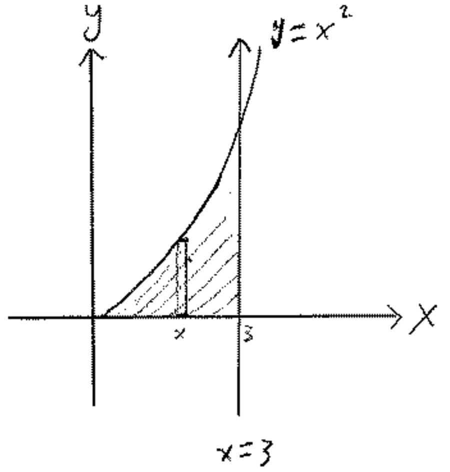
$$V = 2\pi \int_0^3 (3-x)x^2\,dx$$

$$= 2\pi \int_0^3 (3x^2 - x^3)\,dx$$

$$= 2\pi \left[x^3 - \frac{x^4}{4}\right]_0^3$$

$$= 2\pi\left[27 - \frac{81}{4}\right] = 2\pi\left[\frac{108-81}{4}\right]$$

$$= \frac{27}{2}\pi$$



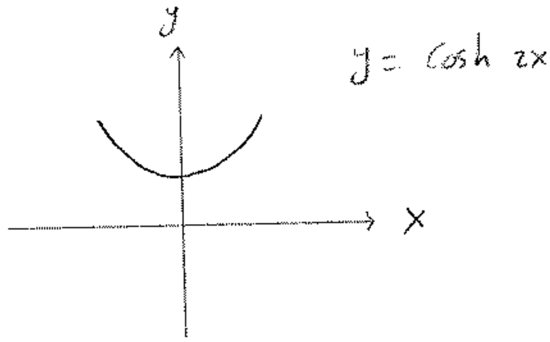
4. $\lim\limits_{x \to -\infty} \cosh\ 2x =$

(a) 0

(b) 1

(c) $+\infty$

(d) $-\infty$



5. $\tanh(\ln x) =$

(a) $\dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

(b) $\dfrac{x^2 + 1}{x^2 - 1}$

(c) $\dfrac{x^2 - 1}{x^2 + 1}$

(d) $\dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\tanh(\ln x) = \frac{e^{\ln x} - e^{-\ln x}}{e^{\ln x} + e^{-\ln x}}$$

$$= \frac{x - \frac{1}{x}}{x + \frac{1}{x}}$$

$$= \frac{x^2 - 1}{x^2 + 1}$$

6. $\int \ln x \, dx =$

put $u = \ln x$, $dv = dx$

$du = \frac{1}{x} dx$, $v = x$

$\int u \, dv = uv - \int v \, du$

$\int \ln x \, dx = x \ln x - \int dx$

$= x \ln x - x + C$

(a) $x \ln x + \frac{1}{x} + c$

(b) $x \ln x + c$

(c) $\frac{1}{x} + c$

(d) $x \ln x - x + c$

7. $\int (\cosh x - \sinh x)^{14} \, dx =$

$\cosh x - \sinh x = e^{-x}$

$(\cosh x - \sinh x)^{14} = e^{-14x}$

$\int (\cosh x - \sinh x)^{14} dx = \int e^{-14x} dx$

$= -\frac{1}{14} e^{-14x} + C$

(a) $\frac{1}{15}(\cosh x - \sin x)^{15} + c$

(b) $-\frac{1}{15} e^{15x} + c$

(c) $\frac{1}{14} e^{14x} + c$

(d) $-\frac{1}{14} e^{-14x} + c$

8. The area of the surface generated by revolving the parametric curve:

$x = \sin^2 t, \; y = \sin t \cos t, \; 0 \le t \le \frac{\pi}{2}$, about the x-axis is:

(a) 3π

(b) 2π

(c) π

(d) $\frac{\pi}{2}$

$L = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$

$\frac{dx}{dt} = 2 \sin t \cos t$, $\left(\frac{dx}{dt}\right)^2 = 4 \sin^2 t \cos^2 t$

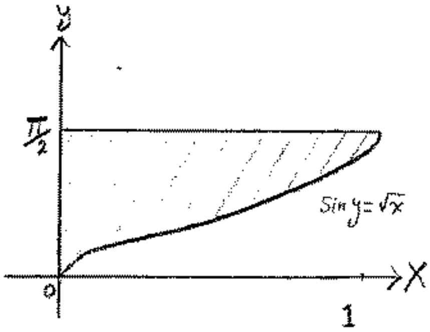
$\frac{dy}{dt} = \cos^2 t - \sin^2 t$, $\left(\frac{dy}{dt}\right)^2 = \cos^4 t - 2 \cos^2 t \sin^2 t + \sin^4 t$

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (\sin^2 t + \cos^2 t)^2 = 1$

$L = \int_0^{\frac{\pi}{2}} 2\pi \sin t \cos t \, dt = 2\pi \left. \frac{\sin^2 t}{2} \right]_0^{\frac{\pi}{2}} = \pi$.

Solve each of the following questions:

9. Consider the region bounded above by the line $y=\frac{\pi}{2}$ and below by the curve $\sin y=\sqrt{x}$ as shown.
Suppose we rotate the region about the x-axis to make a solid, and we want the volume.



(a) Set up an integral to find the volume using washers method.

$\sin y=\sqrt{x} \Rightarrow y=\sin^{-1}\sqrt{x} \Rightarrow f(x)=\pi/2\ ,\ g(x)=\sin^{-1}\sqrt{x}$

$$V=\int_a^b \pi\left[(f(x))^2-(g(x))^2\right]dx$$

$$\Rightarrow \quad V=\int_0^1 \pi\left[\left(\tfrac{\pi}{2}\right)^2-\left(\sin^{-1}\sqrt{x}\right)^2\right]dx$$

(b) Set up an integral to find the volume using shells method.

$\sin y=\sqrt{x} \Rightarrow x=\sin^2 y=f(y)$

$$V=\int_c^d 2\pi y\, f(y)\, dy = \int_0^{\pi/2} 2\pi y \sin^2 y\, dy .$$

10. Find $\int\sqrt{\coth^5 x}\ \operatorname{csch}^2 x\,dx$

$$=\int (\coth x)^{5/2}\operatorname{csch}^2 x\,dx$$

Put $u=\coth x \Rightarrow du=-\operatorname{csch}^2 x\,dx$

$$\Rightarrow \int (\coth x)^{5/2}\operatorname{csch}^2 x\,dx = -\int u^{5/2}\,du = -\frac{2}{7}u^{7/2}+C$$

$$= -\frac{2}{7}\coth^{7/2} x + C$$

11. Evaluate $\int_0^{\frac{\pi}{3}} x \sec^2\left(\frac{x}{2}\right) dx$

Using integration by parts, we put

$u = x \quad , \quad dv = \sec^2 \frac{x}{2}\, dx$

$du = dx \quad , \quad V = 2 \tan \frac{x}{2}$

$\int u\,dv = uv - \int v\,du$

$\int x \sec^2 \frac{x}{2}\, dx = 2x \tan \frac{x}{2} - \int 2 \tan \frac{x}{2}\, dx$

$= 2x \tan \frac{x}{2} - 2\int \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} dx$

$= 2x \tan \frac{x}{2} - 4\int \frac{\frac{1}{2}\sin \frac{x}{2}}{\cos \frac{x}{2}} dx$

$= 2x \tan \frac{x}{2} + 4 \ln\left|\cos \frac{x}{2}\right| + C$

$\int_0^{\frac{\pi}{3}} x \sec^2 \frac{x}{2}\, dx = \left[2x \tan \frac{x}{2} + 4 \ln\left|\cos \frac{x}{2}\right|\right]_0^{\frac{\pi}{3}}$

$= \frac{2\pi}{3} \tan \frac{\pi}{6} + 4 \ln\left|\cos \frac{\pi}{6}\right| - [0 + 4\ln 1]$

$= \frac{2\pi}{3} \frac{1}{\sqrt{3}} + 4 \ln\left(\frac{\sqrt{3}}{2}\right) - 0$

$= \frac{2\pi}{3\sqrt{3}} + 4 \ln\left(\frac{\sqrt{3}}{2}\right)$

12. Find the arc length of the curve

$y = \frac{1}{3}\left(x^2+2\right)^{\frac{3}{2}}$ from $x = 0$ to $x = 3$

$f(x) = \frac{1}{3}(x^2+2)^{\frac{3}{2}}$

$f'(x) = \frac{1}{2}(x^2+2)^{\frac{1}{2}}\, 2x$

$= x\,(x^2+2)^{\frac{1}{2}}$

$[f'(x)]^2 = x^2(x^2+2) = x^4 + 2x^2$

$1 + [f'(x)]^2 = 1 + x^4 + 2x^2 = (x^2+1)^2$

$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_0^3 \sqrt{(x^2+1)^2}\, dx$

$= \int_0^3 (x^2+1)\, dx$

$= \left[\frac{1}{3}x^3 + x\right]_0^3$

$= 9 + 3$

$= 12$