King Fahd University of Petroleum and Minerals

Department of Mathematical Sciences

Math 101 (calculus I)

Quiz 7 (B) Semester I, 2004-2005 (041)

Name:..... **ID #**..... **Sec#**.....

(1) Let $f(x) = 2x + 3x^{\frac{2}{3}}$. Find

(a) x-intercepts and y-intercepts.

$y=0 \Rightarrow 2x+3x^{\frac{2}{3}}=0 \Rightarrow x(2+3x^{-\frac{1}{3}})=0 \Rightarrow x=0$ or $x=-\frac{27}{8}$

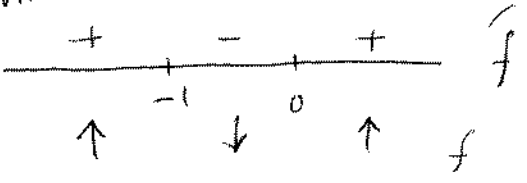
$x=0 \Rightarrow y=0$ $\therefore$ intercepts are $(0,0)$, $(-\frac{27}{8},0)$

(b) The intervals on which f is increasing and and the intervals on which f is decreasing.

$f'(x) = 2+2x^{-\frac{1}{3}} = 2\left(1+\frac{1}{\sqrt[3]{x}}\right)$

$f'(x)=0 \Rightarrow 1+\frac{1}{\sqrt[3]{x}}=0 \Rightarrow \frac{1}{\sqrt[3]{x}}=-1 \Rightarrow x=-1$

Critical points are: $-1, 0$



(c) Relative extrema

$(-1,1)$ relative max

$(0,0)$ relative min.

(d) The open intervals on which f is concave up and on which f is concave down.

$f''(x) = -\frac{2}{3}x^{-\frac{4}{3}} = -\frac{2}{3x^{\frac{4}{3}}} < 0 \quad \forall x,\ x\neq 0.$

The graph of f is concave down

(e) x-coordinates of all inflection points

$$f''(x) = -\frac{2}{3x^{\frac{4}{3}}} \neq 0 \Rightarrow \text{No inflection points.}$$

(f) the point of vertical tangency and cups (if any).

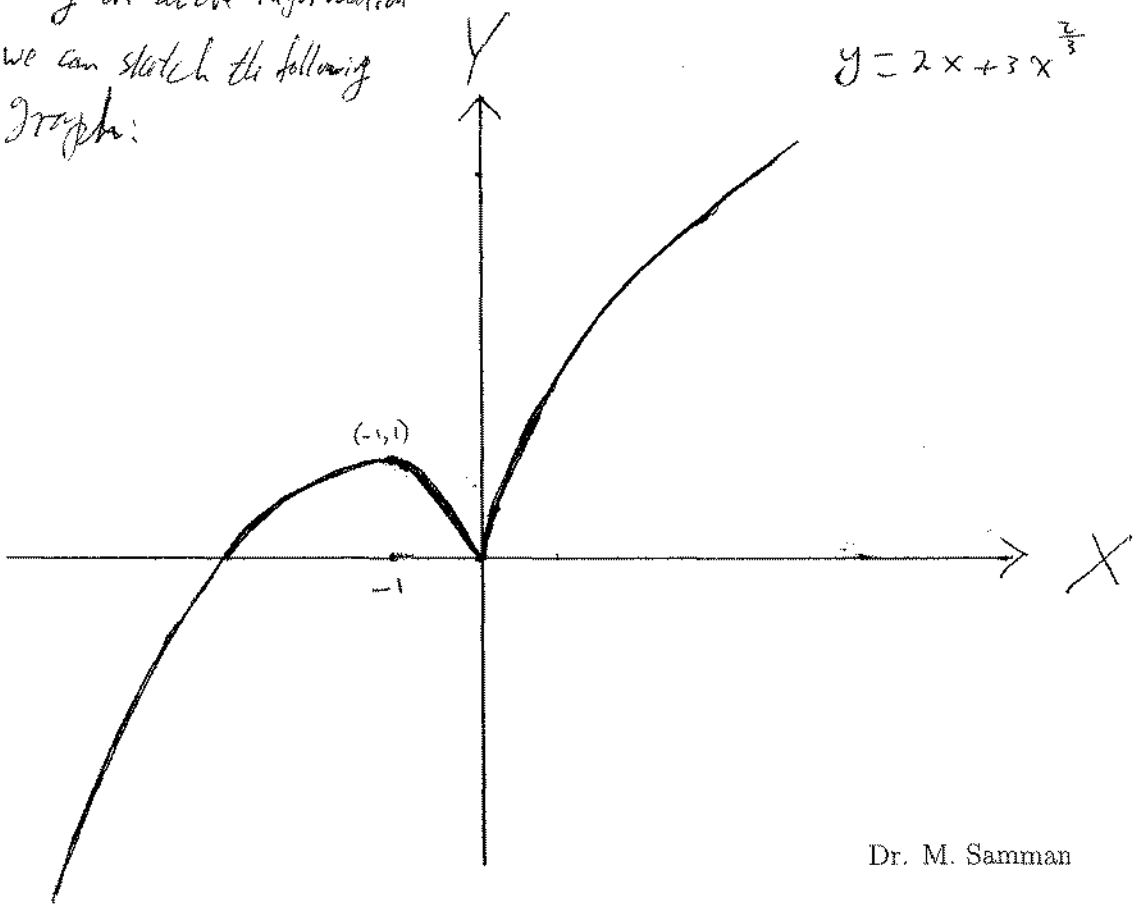
$$f'(x) = 2\left(1 + \frac{1}{\sqrt[3]{x}}\right)$$

$f'(0)$ does not exist and $\lim_{x \to 0^+} f'(x) = +\infty$, $\lim_{x \to 0^-} f'(x) = -\infty$

$\Rightarrow$ There is a vertical tangent line and a cusp at $x = 0$

(g) Sketch the graph

Using the above information we can sketch the following graph:



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