King Fahd University of Petroleum and Minerals

Department of Mathematical Sciences

Math 101 (calculus I)

Quiz 7 (A) Semester I, 2004-2005 (041)

Name:..... **ID #**..... **Sec#**.....

(1) Let $f(x) = x^{\frac{4}{3}} - x^{\frac{1}{3}}$. Find

(a) x-intercepts and y-intercepts.

$y=0 \Rightarrow x^{\frac{4}{3}} - x^{\frac{1}{3}} = 0 \Rightarrow x^{\frac{1}{3}}(x-1) = 0 \Rightarrow x = 0, \; x = 1$

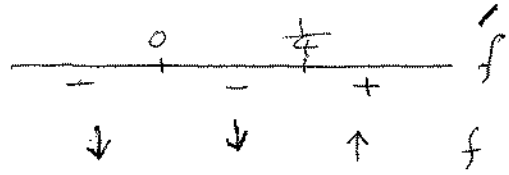
$x = 0 \Rightarrow y = 0$. $\therefore$ intercepts : $(0,0)$, $(1,0)$

(b) The intervals on which f is increasing and and the intervals on which f is decreasing.

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} - \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3}x^{-\frac{2}{3}}\left[4x - 1\right] = \frac{4x-1}{3x^{\frac{2}{3}}}$

$f'(x) = 0 \Rightarrow \frac{1}{3}x^{-\frac{2}{3}}(4x-1) = 0 \Rightarrow x = \frac{1}{4}$

Critical points are: $x = 0, \; x = \frac{1}{4}$

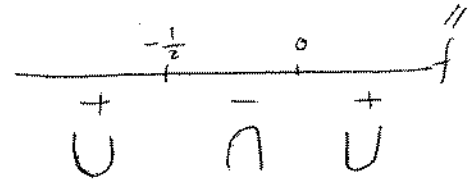


(c) Relative extrema

$\left(\frac{1}{4}, \frac{-3}{a\sqrt[3]{4}}\right)$ relative min

(d) The open intervals on which f is concave up and on which f is concave down.

$f''(x) = \frac{4}{9}x^{-\frac{2}{3}} + \frac{2}{9}x^{-\frac{5}{3}} = \frac{2}{9}x^{-\frac{5}{3}}\left[2x + 1\right] = \frac{2}{9}\frac{(2x+1)}{x^{\frac{5}{3}}}$



(e) x-coordinates of all inflection points

$f''(x) = 0 \Rightarrow 2x+1 = 0 \Rightarrow x = -\frac{1}{2}$. Also $x = 0$ is an inflection pt.

(f) the point of vertical tangency and cups (if any).

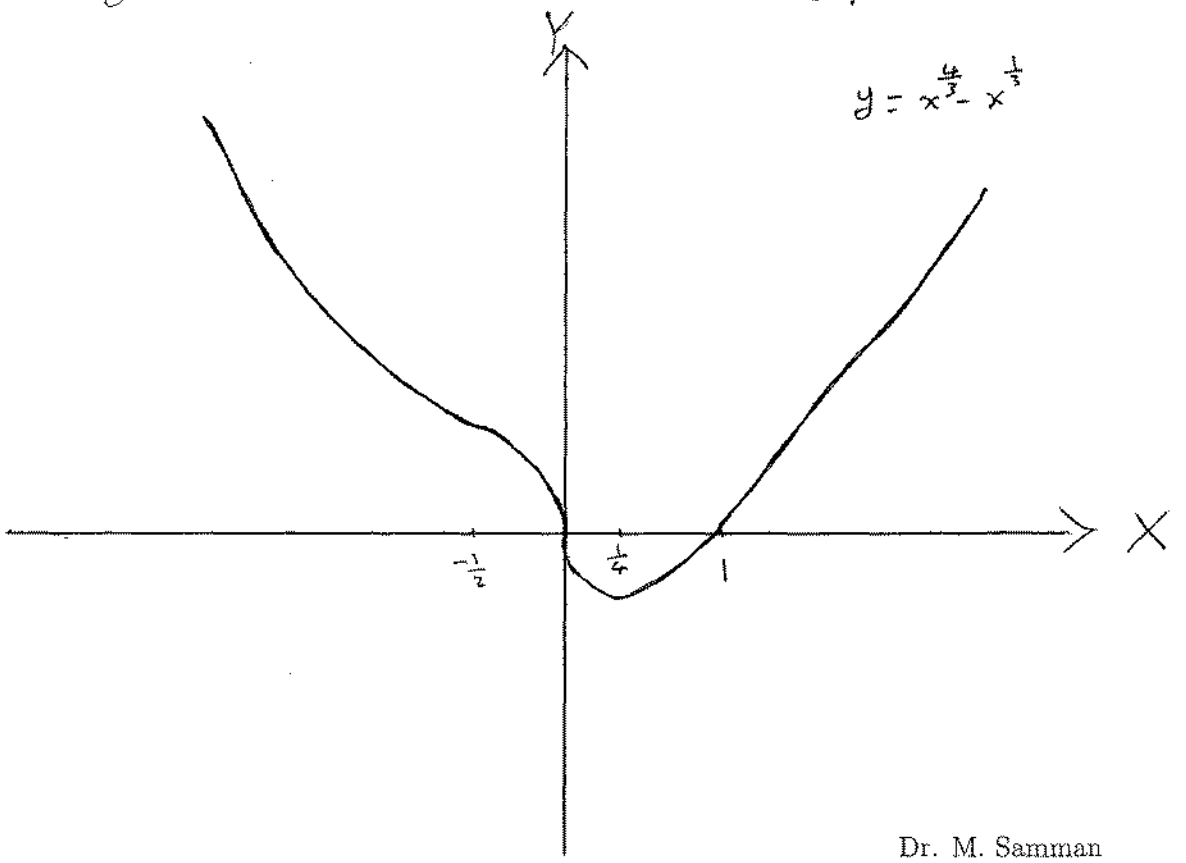
$f'(x) = \frac{4x-1}{3x^{\frac{2}{3}}}$

$f'(0)$ does not exist and $\lim_{x \to 0^+} f'(x) = -\infty$, $\lim_{x \to 0^-} f'(x) = -\infty$

$\Rightarrow$ There is a vertical tangent line at $x = 0$

(g) Sketch the graph

Using the above information, we can sketch the graph as:



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