King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 201-11&14 Quiz2(a) Fall 2006(061)

ID#:_____ NAME: MRF

Serial#_____ Sec: Key

(1) Show that $f(x,y) = \begin{cases} \frac{\tan(x^2+y^2)}{x^2+y^2} & (x,y) \neq (0,0) \\ 1 & (x,y) = (0,0) \end{cases}$ is continuous at $(0,0)$

$$\lim_{(x,y)\to(0,0)} \frac{\tan(x^2+y^2)}{x^2+y^2} = \lim_{r^2\to 0} \frac{\tan(r^2)}{r^2} \quad \text{L.R} \left(\frac{0}{0}\right)$$

$$= \lim_{r\to 0} \frac{2r \sec^2 r^2}{2r} = \boxed{1}$$

$$= f(0,0)$$

$\therefore$ f is continuous at $(0,0)$

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Math 201-11&14 Quiz2(b) Fall 2006(061)

ID#:_____ NAME: MRF

Serial#_____ Sec: K-4

(1) (a) Evaluate $\lim_{(x,y,z)\to(0,0,0)} \frac{xz}{x^2+2y^2+z^2}$

(b) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $(x+y)^2 = (y-z)^2$

on x-axis y=0=z

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xz}{x^2+2y^2+z^2} = \lim_{(x,0,0)\to(0,0,0)} \frac{0}{x^2} = 0 \quad (2)$$

on x=z, y=0

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xz}{x^2+2y^2+z^2} = \lim_{(x,0,x)\to(0,0,0)} \frac{x^2}{x^2+0+x^2} = \frac{1}{2} \quad (3)$$

$$\Rightarrow \lim_{(x,y,z)\to(0,0,0)} \frac{xz}{x^2+2y^2+z^2}$$ D.N.E

(b) $2(x+y) = 2(y-z)\cdot -z_x$

$$\Rightarrow z_x = \frac{(x+y)}{z-y} \quad (2)$$

$2(x+y) = 2(y-z)\cdot(1-z_y)$

$= 2(y-z) - 2(y-z)z_y$

$\Rightarrow 2(y-z)z_y = 2(y-z) - 2(x+y)$

$$z_y = \frac{(y-z)-(x+y)}{y-z} = \frac{-z-x}{y-z} = \frac{z+x}{z-y} \quad (3)$$