King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 102(12&17) Exam1(A) Spring 2004(042)

ID#:_____ NAME:_____

Sec#_____ Serial#_____

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(1) Use the definition of the area, $A = \lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$ with x_k^* as the right end point of each subinterval, to evaluate $\int_0^3 (x^2-6x)dx$. (10pts)

(2) Evaluate each of the following integrals, if it exists. (28pts)

(i) $\int \tan x \ln(\cos x) dx$.

(ii) $\int_{-2}^{1} \frac{1}{x^4} dx$.

(iii) $\int \frac{x}{\sqrt{x+1}} dx$.

(iv) $\int_{-1}^{1} \frac{\sin x}{1+x^2} dx$.

(v) $\int \frac{\sec x \tan x}{1+sec^2 x} dx$.

(vi) $\int_0^{\pi} \sqrt{\sin(x) - \sin^3(x)}dx$.

(vii) $\int_{-3}^{1} f(x)dx$, where f(x)=$\begin{cases} (\text{-x-1})^2 & \text{if } -3 \leq x \leq 0 \\ \text{-}\sqrt{1-x^2} & \text{if } 0 \leq x \leq 1. \end{cases}$

(3) (a) Estimate the value of the integral $\int_1^3 \sqrt{x^2+3}dx$. (10pts)

(b) Solve the initial value problem: $\frac{dy}{dx} = 3\sin 2x + 6$, $y(0) = 1$. (6pts)

(c) (i) State the mean value theorem for integrals. (ii) If f is continuous and $\int_1^3 f(x)dx = 8$, show that f equal to 4 at least once on the interval $[1, 3]$. (10pts)

(4) Evaluate $\lim_{n\to+\infty} \frac{1}{n}[(\frac{1}{n})^9+(\frac{2}{n})^9+(\frac{3}{n})^9+...+(\frac{n}{n})^9]$ (6pts)

(5) Find $\lim_{h\to 0} \frac{1}{h} \int_3^{3+h} \sqrt{t^3-2}dt$. (8pts)

(6) Find the area of the region bounded by $y=\sqrt{x}$ and $y=\frac{1}{2}x$ on the interval $[0,9]$.(10pts)

(7) Mark each of the following as True or False [T or F]. (Show your work) (12pts)

(i) If $\int_0^9 f(x)dx = 4$, then $\int_0^3 xf(x^2)dx = \frac{4}{3}$.

(ii) If f is continuous on $[0,1]$, then $\int_0^1 xf(x)dx = x\int_0^1 f(x)dx$.

(iii) If f' is continuous on $[a,b]$, then $\int_a^b f'(x)dx = f(b) - f(a)$.

(iv) $\frac{d}{dx}\int_e^{e^2} \ln(t)dt = \ln e^2 - \ln e = 2 - 1 = 1$

(v) $\sum_{k=1}^{n-1} k^2 = \frac{n(n+1)(2n+1)}{2} - n^2$

(vi) If $\int_{g(a)}^{g(b)} f(x)dx = 8$, then $\int_a^b f(g(x))g'(x)dx = g(8)$.

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(1) Use the definition of the area, $A = \lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$ with x_k^* as the right end point of each subinterval, to evaluate $\int_0^2 (4x - x^2)dx$. (10pts)

(2) Evaluate each of the following integrals, if it exists. (28pts)

(i) $\int \cot x \ln(\sin x) dx$.

(ii) $\int_{-3}^{3} \frac{1}{x^2} dx$.

(iii) $\int \frac{2x}{\sqrt{x+2}} dx$.

(iv) $\int_{-1}^{1} \frac{\tan x}{1+x^2} dx$.

(v) $\int \frac{-\csc x \cot x}{1+csc^2 x} dx$.

(vi) $\int_0^{\pi} \sqrt{\sin(x) - \sin^3(x)}dx$.

(vii) $\int_{-3}^{1} f(x)dx$, where f(x)=$\begin{cases} (\text{-x-1})^2 & \text{if } -3 \leq x \leq 0 \\ \text{-}\sqrt{1-x^2} & \text{if } 0 \leq x \leq 1. \end{cases}$

(3) (a) Estimate the value of the integral $\int_0^2 \sqrt{x^3+1}dx$. (10pts)

(b) Solve the initial value problem: $\frac{dy}{dx} = 2\sin 3x + 6$, $y(0) = 1$. (6pts)

(c) (i) State the mean value theorem for integrals. (ii) If f is continuous and $\int_1^3 f(x)dx = 6$, show that f equal to 3 at least once on the interval $[1, 3]$. (10pts)

(4) Evaluate $\lim_{n\to+\infty} \frac{1}{n}[(\frac{1}{n})^9+(\frac{2}{n})^9+(\frac{3}{n})^9+...+(\frac{n}{n})^9]$ (6pts)

(5) Find $\lim_{x\to 2} \frac{1}{x-2} \int_2^x \sqrt{t^3-2}dt$. (8pts)

(6) Find the area of the region bounded by $y=\sqrt{x}$ and $y=\frac{1}{2}x$ on the interval $[0,10]$.(10pts)

(7) Mark each of the following as True or False [T or F]. (Show your work) (12pts)

(i) If $\int_0^9 f(x)dx = 4$, then $\int_0^3 xf(x^2)dx = 2$.

(ii) If f is continuous on $[1, 2]$, then $\int_1^2 xf(x)dx = x\int_1^2 f(x)dx$.

(iii) If f' is continuous on $[a, b]$, then $\int_b^a f'(x)dx = f(b) - f(a)$.

(iv) $\frac{d}{dx}\int_e^{e^3} \ln(t)dt = \ln e^3 - \ln e = 3 - 1 = 2$

(v) $\sum_{k=1}^{n-1} k^3 = (\frac{(n-1)n}{2})^2$

(vi) If $\int_{g(a)}^{g(b)} f(x)dx = 8$, then $\int_a^b f(g(x))g'(x)dx = 8$.

Dr. M. R. Alfuraidan