

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 101 (Calculus I)
Major Exam # 1
Semester I, 2004-2005 (041)

Name: _____

Section #: _____

ID #: _____

- Show complete work for full credit.
- Use of graphic calculators and mobile phones or any other equipment is not allowed in this exam.

Question	Score
1	
2	
3	
4	
Total:	

1. Find each of the following limits, if it exists.

(5 points each)

$$(a) \lim_{x \rightarrow 3} (x + 2) \cdot \frac{3 - x}{|3 - x|}.$$

$$(b) \lim_{x \rightarrow 2} \frac{x^4 - 16}{x^2 - x - 2}.$$

$$(c) \lim_{h \rightarrow 0} \frac{\sqrt{2 + h} - \sqrt{2}}{h}.$$

$$(d) \lim_{x \rightarrow -\infty} (-3x + 1)^3(2x + 1)^2(x + 1).$$

$$(e) \lim_{x \rightarrow 0^+} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right).$$

$$(f) \lim_{x \rightarrow 0} \frac{\sin x - 7x}{x \cos x}.$$

$$(g) \lim_{x \rightarrow +\infty} \frac{2x + x \sin 3x}{5x^2 - 2x + 1}.$$

$$(h) \lim_{x \rightarrow 3} \frac{\sin(\pi x)}{x - 3}. \quad (\text{Hint: let } t = \pi x - 3\pi)$$

$$(i) \lim_{x \rightarrow +\infty} \frac{\sin x}{x}. \quad (\text{Hint: Use the Squeezing Theorem})$$

$$(j) \lim_{x \rightarrow +\infty} \sqrt[3]{\frac{8 + x^2}{x(x + 1)}}.$$

2. (a) Use the limit definition to prove that

$$\lim_{x \rightarrow 15} \sqrt{x+1} = 4.$$

(5 points)

- (b) Find all asymptotes for the graph of

$$f(x) = \frac{4x - 3}{\sqrt{x^2 + 1}}.$$

(12 points)

- (c) Prove that the equation

$$x^5 - 3x^4 - 2x^3 - x + 1 = 0$$

has a solution between 0 and 1.

(5 points)

3. (a) Find all numbers at which f is discontinuous, where

$$f(x) = \begin{cases} -x^2 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ -\frac{1}{x} & \text{if } x > 1 \end{cases}$$

(8 points)

- (b) Show that $f(x) = \sqrt{16 - x}$ is continuous on the interval $(-\infty, 16]$. (5 points)

- (c) Suppose that f is a continuous function, $\lim_{x \rightarrow 4} f(x) = 0$ and $\lim_{x \rightarrow 4} g(x) = -3$. Find

$$\lim_{x \rightarrow 4} \frac{xg(x)}{\sqrt{f(x)} + 1}. \quad (5 \text{ points})$$

(d) Consider the function $f(x) = x^2 + 1$ and the point $P(2, f(2))$.

(i) Find the slope of the graph of $y = f(x)$ at the point P .

(ii) Find the instantaneous rate of change of y with respect to x at the general point $x = x_0$.

(10 points)

4.

Extra Credits (10 points)

(a) If $\lim_{x \rightarrow a} [f(x) + g(x)] = 4$ and $\lim_{x \rightarrow a} [f(x) - g(x)] = 1$, find $\lim_{x \rightarrow a} f(x)g(x)$.

(b) Evaluate $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{\sqrt{x} + 1}$.