

King Fahd University of Petroleum and Minerals  
Department of Mathematical Sciences

Semester(Term 041)

**MATH 101**

**Second Major Exam**

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Name: \_\_\_\_\_ ID# \_\_\_\_\_

Section: \_\_\_\_\_

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9	
Total	

1. Use the limit definition of the derivative to compute the derivative of  $f(x) = \frac{2}{\sqrt{x}}$ .

(8 points)

2. Assume that

$$f(x) = \begin{cases} \frac{1}{4}x^3 - \frac{1}{2}x^2, & \text{if } x \geq 2 \\ \frac{-6x - 6}{x^2 + 2}, & \text{if } x < 2 \end{cases}$$

Determine if  $f$  is differentiable at  $x = 2$ , i.e. determine  $f'(2)$  if it exists.

(10 points)

3. For each of the following functions, compute the derivative. Show all work including each step in your derivation.

(5 points each)

(a)  $y = \left(\frac{x}{4} + x^{-5}\right)^{\frac{1}{2}}.$

(b)  $y = \left(\frac{8x - x^6}{x^3}\right)^{-\frac{4}{5}}.$

(c)  $y = \sec^2(x^4) \tan^3(x^4).$

$$(d) y = \sqrt{\sin(7x + \cos 5x)}.$$

$$(e) y = \tan^3 \sqrt{\cot 7x}.$$

$$(f) y = \frac{1}{x\sqrt{x^2 + 1}}.$$

4. The point  $(1, 2)$  lies on the hyperbola with equation  $x^2 + 2xy - y^2 + x = 2$ . Using implicit differentiation, determine the equation of the tangent line to the hyperbola at  $(1, 2)$ .

(9 points)

5. Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

(9 points)

6. Estimate  $\sqrt{3.9}$ . Show all work. (**Hint:** Use local linear approximation)

(9 points)

7. Let  $f(x) = 4x - \sin 3x$ ,  $-\frac{\pi}{6} < x < \frac{\pi}{6}$ . (9 points)

- (a) Show that  $f(x)$  is a one-to-one function.
- (b) Show that  $f^{-1}(x)$  is differentiable on the interval  $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$ .
- (c) Find a formula for the derivative of  $f^{-1}$ .

8. Solve for  $x$ ,  $\log_{e^2}^{(7-x)^2} + \ln(3x + 5) = \ln(24x)$ .

(8 points)

9. Find the inverse of  $f(x) = 3 - e^{x-2}$ .

(8 points)