

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences

(Term 041)

MATH 101

Final Examination

Time: 3 hours

Name: _____

ID# _____

Sec.#: _____

No Calculator is Allowed in the Exam

Show All Necessary Work

Note:

For Part II and Part III you should write your answers in the boxes on the answer sheet provided on the next page.

Part - I

1. State the mean value theorem, then apply it to find the value(s) of c in the interval $[0, 2]$ that satisfies the assertion of the theorem for the function

$$f(x) = x^3 - x^2 - x + 1.$$

2. Find the limit of each one of the following:

$$(a) \lim_{x \rightarrow -\infty} \frac{-x}{\sqrt{16 + x^2}}$$

$$(b) \lim_{x \rightarrow \frac{\pi}{2}} \frac{e^{\cos x} - 1}{x - \frac{\pi}{2}}$$

$$(c) \lim_{x \rightarrow 5^-} \frac{x^2 - 3x - 10}{x^2 - 25}$$

$$(d) \lim_{x \rightarrow 2} \frac{1}{(x - 2)^3}$$

$$(e) \lim_{x \rightarrow \infty} \left(\frac{x}{x + 1} \right)^x$$

3. Given the function

$$f(x) = \begin{cases} 2x - 2 & \text{if } x < -1 \\ Bx + C & \text{if } -1 \leq x \leq 1 \\ 5x + 7 & \text{if } x > 1 \end{cases}$$

Determine the constants B and C so that the function f is continuous for all real values of x .

4. (a) Find the derivative y' for each of the following:

(i) $y = 4 \tan^{-1}(2e^x)$

(ii) $y = 3^{\sec x^2} + \cot^3 5x$

(iii) $\cos^{-1}(\sqrt{xy}) = y$

(iv) $y = x^{\sqrt{x}}$

(b) If $\frac{d}{dx} [f(5 - x^2)] = \frac{d}{dx} [5 - x^4]$, find $f'(5 - x^2)$.

5. (a) Let $f(x) = \frac{2x^2}{9 - x^2}$.

Find each one of (i) up to (vii), but do not sketch the graph of f .

(i) x -intercept(s):

y -intercept(s):

(ii) Horizontal asymptote(s):

(iii) Vertical asymptote(s):

(iv) Critical point(s) of $f(x)$:

(v) The intervals where $f(x)$ decreasing or increasing:

(vi) Relative extrema:

(vii) Concavity of the graph of $f(x)$:

(b) Use the following information for $y = g(x)$ and sketch its graph accordingly:

* intercepts: $(2,0)$, $(4,0)$, $(0,2)$

* vertical asymptote: $x = 8$

* decreasing on: $(-\infty, 3) \cup (8, \infty)$, increasing on $(3, 8)$

* inflection point: $(6, 2)$

* relative minimum and cusp : $(3, -2)$

* concave up on $(6, 8) \cup (8, \infty)$, concave down on $(-\infty, 3) \cup (3, 6)$

* $\lim_{x \rightarrow +\infty} g(x) = 0$

6. A particle is moving along a coordinate line according to the position function

$S(t) = e^{-4t} \sin 3t$, where S in meters and t in seconds.

(a) Find the velocity of the particle at time $t = 2$.

(b) Find the position of the particle at $t = 0$.

(c) Find the acceleration of the particle at $t = 2$.

7. Use the definition of derivative to prove that

$$\frac{d}{dx} [\log_a x] = \frac{1}{x \ln a}, \quad x > 0$$

8. The dome of a mosque has the shape of semi-sphere. Its radius is measured to be 320 cm with a possible measurement error of ± 0.02 cm. Use differentials to estimate:

(a) the relative error in the computed surface area.

(b) the percentage error in the computed surface area.

9. Use Newton's method to approximate the real solution of the equation $x^3 + 5x - 3 = 0$ to two decimal places. (Hint: you may start with $x_1 = 0$)

10. A company wishes to manufacture a box with a volume of 36 cubic feet that is open on top and that is twice as long as it is wide. Find the dimensions of the box produced from the minimum amount of material.

Part - II

Circle the right answer in each of the following:

11. If $f(x) = \sqrt{x^2 + 7x + 1}$, then $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$

(a) 1

(b) 0

(c) 3

(d) $\frac{3}{2}$

(e) does not exist

12. If $f(x) = x^2g(x)$, $g(3) = 8$ and $g'(3) = 2$, then $f'(3) =$

(a) 0

(b) 16

(c) 18

(d) 48

(e) 66

13. The equation of the tangent line to the graph of the equation $(\frac{x}{4})^2 + (\frac{y}{6})^2 = 2$ at the point $(4, 6)$ is

(a) $9x - 2y = 24$

(b) $x + y = 10$

(c) $5x - 2y = 8$

(d) $3x + 2y = 24$

(e) $3x - y = 6$

14. The function $f(x) = 3(x - 1)^{\frac{2}{3}}$ is

(a) not defined at $x = 1$

(b) having a cusp at $x = 1$

(c) concave up for $x < 1$

(d) concave up on $(-\infty, \infty)$

(e) differentiable everywhere

15. $\lim_{t \rightarrow 0} \frac{|t - 10| - 10}{t}$

(a) 1

(b) 0

(c) -1

(d) ∞

(e) does not exist

Part - III

16. Answer TRUE (\checkmark) or FALSE (X)

(a) $f(x) = \sqrt{x}$ is continuous at $x_0 = 0$ but not differentiable at x_0 . ()

(b) We can apply Rolle's theorem to every function $f(x)$ on the interval $[a, b]$ provided that $f(a) = f(b)$. ()

(c) If $\lim_{x \rightarrow +\infty} f(x) = 7$ then it is not possible to obtain $\lim_{x \rightarrow +\infty} f(x - 1)$ ()

(d) The function $f(x) = \frac{1}{x^3}$ has an absolute maximum value 1 on the interval $[-1, 1]$. ()

(e) The function $g(x) = \sin x$ is decreasing on $\left[-\frac{\pi}{2}, 0\right]$ ()

(f) If f and g are both continuous everywhere then so is $\frac{f}{g}$ ()

(g) If g is a continuous function then it is differentiable ()

(h) If f is differentiable and f^{-1} exists then f^{-1} is differentiable. ()

(i) $g(x) = \frac{x}{x+3}$ is increasing for all x in its domain. ()

(j) If $\lim_{x \rightarrow 2} f(x) = 1$ and $\lim_{x \rightarrow 2} g(x) = \infty$, then it should imply $\lim_{x \rightarrow 2} [f(x)]^{g(x)} = 1$ ()