

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 425 Exam2 Fall 2005(051)

ID#: _____

NAME: _____

(1) (a) How many distinct labelings (from a fixed set of n labels) are there for

(i) C_n $n \geq 3$, (ii) P_n ($n \geq 2$); (iii) $K_{1,n}$ ($n \geq 2$).

(b) Let G be a graph. An edge-automorphism of a nonempty graph G is an edge-isomorphism of G with itself. The set of all edge-automorphisms of G (under composition) forms a group, called the edge-group of G and denoted by $Aut_E(G)$. Find two nonisomorphic graphs that are edge-isomorphic.

- (2) (a) Is there a bipartite graph with degrees $1, 1, 1, 1, 1, 1, 1, 1, 2, 3, 4, 5$.
(b) How many distinct spanning trees can be constructed on 4 labeled vertices.
(c) Count the spanning trees of the graph G below by cofactors.

(3) A connected graph G is minimally connected if for every edge e of G the graph $G - e$ is not connected. Prove that a connected graph is a tree if and only if it is minimally connected.

(b) Characterize those graphs with the property that every connected subgraph is an induced subgraph.

- (4) (a) Find a graph G of diameter 3 for which $\kappa_1(G) \neq \delta(G)$.
(b) Prove that for any graph G $\kappa(G) \leq \kappa_1(G) \leq \delta(G)$.
(c) Let G be a graph with degree sequence d_1, d_2, \dots, d_n , where $d_1 \leq d_2 \leq \dots \leq d_n$. Define $H = G + K_1$. Determine $\kappa_1(H)$.

- (5) (a) Define (i) a flow in a network (ii) a cut in a network
(b) Prove that $Val f \leq Cap f$.