KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 211: BUSINESS STATISTICS I
Semester 042
Major Exam #2
Saturday April 30, 2005

Please **circle** your instructor's name: Marwan Al-Momani
Mohammad F. Saleh
Walid S. Al-Sabah

Name: ID# Section

Question No	Full Marks	Marks Obtained
1	6	
2	6	
3	6	
4	6	
5	6	
6	5	
7	6	
8	9	
Total	50	

1) Answer True or False. *(6 points)* **ONE POINT EACH**

   i) When a market research manager records the number of potential customers who were surveyed indicating that they like the product design, the random variable—number who like the design—is a discrete random variable.
   TRUE

   ii) If the variance of one discrete random variable is 5.0 and the variance of a second discrete random variable is 8.0, the covariance for these two variables will be 13.0.
   FALSE

   iii) The number of defects discovered in a random sample of 100 products is binomially distributed with p = .03. Based on this, the expected number of defects is 3.
   TRUE

   iv) A warehouse contains five parts made by the Stafford Company and three parts made by the Wilson Company. If an employee selects two of the parts from the warehouse at random, the probability that all are from the Wilson Company is 3/28.
   TRUE

   v) If the time it takes for a customer to be served at the cafeteria is thought to be uniformly distributed between 3 and 8 minutes, then the probability that the time it takes for a randomly selected customer will be less than 5 minutes is 0.40.
   TRUE

   vi) If ρ is the correlation coefficient of the linear relation ship between two variables, then $\rho = 0$ means that there is no relationship between the two variables.
   FALSE

2) *(6 points)* As part of marketing analysis, 200 persons were asked the question, "How many times do you go to Al-Rashid Mall weekly?" The numbers of responses were 70, 84, 36, and 10 for 0, 1, 2 and 3 times, respectively. Let X be the number of times a randomly selected person went to the mall weekly. Use the survey data to find:

   i) The probability distribution of X
   ii) The probability that the person will go to the mall more than two times in any week.
   iii) The expected number of persons who go to the mall weekly.

Answer

i. 2 points

x	Freq	P(x)	x.P(x)
0	70	0.35	0
1	84	0.42	0.42
2	36	0.18	0.36
3	10	0.05	0.15
Total	200	1	0.93

ii. **2 points** $P(x > 2) = P(x = 3) = 0.05$

iii. **2 points** $E(x) = \sum xP(x) = 0 + 0.42 + 0.36 + 0.15 = 0.93$

3) *(6 points)* A bank interested in expanding its customer base requested a market survey. Customers were asked to rate overall services on a 3-point scale from excellent to poor. The results are summarized in the following table, where we also include the age of the respondents:

Age	Rating			Total
	Excellent	Average	Poor	
Under 35	*120*	*80*	*40*	*240*
35 or more	*180*	*120*	*60*	*360*
Total	*300*	*200*	*100*	*600*

If one customer is selected at random,

i) Find the probability that his/her rating was excellent.
ii) What is the probability that his/her age is 35 or more?
iii) Are the events in part (i) and (ii) independent? Explain
iv) If the respondent gave a poor rating, what is the probability that he/she was under 35?

Answer:

i. $P(Excellent) = \frac{300}{600} = \frac{1}{2}$ **1 point**

ii. $P(age\ is\ 35 or\ more) = \frac{360}{600} = 0.6$ **1 point**

iii. $Is\quad P(Excellent\ and\ age\ is\ 35\ or\ more) \overset{?}{=} P(Excellent)P(age\ is\ 35\ or\ more)$

$$P(Excellent\ and\ age\ is\ 35\ or\ more) = \frac{180}{600} = 0.3$$

$$P(Excellent)P(age\ is\ 35\ or\ more) = 0.5 \times 0.6 = 0.3$$

YES the two events are independent **3 points**

iv. $P(Under\ 35 | Poor) = \frac{P(Under \cap Poor)}{P(Poor)} = \frac{40/600}{100/600} = \frac{40}{100} = 0.4$ **1 point**

4) *(6 points)* If A and B are two events in the sample space S, where $P(A)=0.50$, $P(B)=0.40$ and $P(A\;or\;B)=0.70$,

i. $P(A|\bar{B})=$

ii. Are the events A and B independent? Note that a yes or no answer is not acceptable unless justified.

Answer

i. $P(A|\bar{B})=\dfrac{P(A\cap\bar{B})}{P(\bar{B})}=\dfrac{P(A)-P(A\cap B)}{1-P(B)}$

To find $P(A\cap B)$

$$P(A\;or\;B)=P(A)+P(B)-P(A\cap B)$$

$$0.7=\;0.4+\;0.5\;-P(A\cap B)\Rightarrow P(A\cap B)=0.9-0.7=0.2$$

So

$$P(A|\bar{B})=\frac{P(A\cap\bar{B})}{P(\bar{B})}=\frac{P(A)-P(A\cap B)}{1-P(B)}=\frac{0.5-0.2}{1-0.4}=\frac{0.3}{0.6}=0.5$$ **3 points**

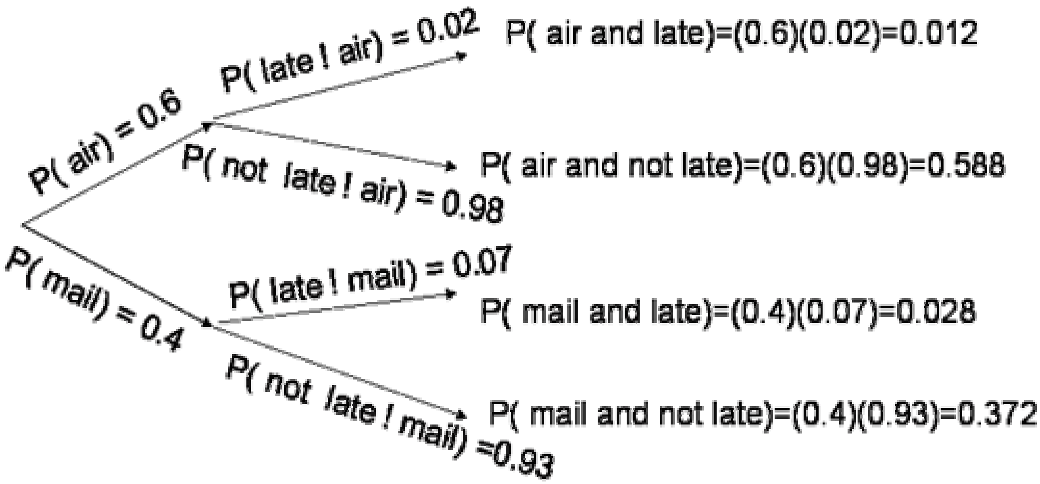
ii.

$$Is\quad P(A\cap B)\overset{?}{=}P(A)P(B)$$

$$0.2=0.4*0.5=0.2$$ **3 points**

So the events A and B are independent

5) *(6 points)* A company sends *60%* of its orders by air, and those deliveries arrive late *2%* of the time. It sends the other *40%* by regular mail, and those arrive late *7%* of the time. Given that the order was delivered late find the probability that it was sent by air.



4 points

$$P(air|late)=\frac{P(air\cap late)}{P(late)}=\frac{P(air)P(late|air)}{P(air)P(late|air)+P(reg.)P(late|reg.)}$$

$$=\frac{0.6(0.02)}{0.6(0.02)+0.4(0.07)}=\frac{0.012}{0.012+0.028}=\frac{0.012}{0.04}=0.3$$ **2 points**

6) *(5 points)* At a checkout counter customers arrive according to a Poisson process at an average rate of 1.5 per minute.
   i) Find the probability that 3 or 4 customers will arrive in a one minute interval.
   ii) Find the probability that at least 3 customers will arrive between 10:00 am and 10:02 am.

Answer:

i. $P(3\ or\ \ 4 \text{ customers arrive in one minute}) = P(3 \text{ customers}) + P(4 \text{ customers})$

$$= \frac{e^{-1.5}(1.5)^3}{3!} + \frac{e^{-1.5}(1.5)^4}{4!} = 0.1255 + 0.0471 = 0.17256$$ **2 points**

ii. $P(\text{at least 3 cuctomers in 2 minutes}) = 1 - P(\text{at most 2 cuctomers in 2 minutes})$

$$= 1 - \sum_{x=0}^{2} \frac{e^{-3}(3)^x}{x!} = 1 - \left[\frac{e^{-3}(3)^2}{2!} + \frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^0}{0!}\right] = 1 - e^{-3}[4.5+3+1]$$ **3 points**

$$= 1 - 8.5e^{-3} = 1 - 0.4232 = 0.5768$$

7) *(6 points)* When circuit boards used in the manufacture of compact disc players are tested, the long run percentage of defectives is 5%. Let $X =$ number of defective boards in a random sample of size $n = 35$.
   i) What is the probability that none of the 35 boards are defective?
   ii) What is the probability that at least 33 of the boards are non defective?
   iii) Calculate the expected value and standard deviation of X.

Answer:

Let X is number of defective, $n = 35,\ p = 0.05$

i. $P(none\ is\ defective) = P(x = 0) = C_0^{35}(0.05)^0(0.95)^{35} = 0.1661$

2 points

ii. let Y is number of non defective $n = 35,\ p = 0.95$

$P(at\ least\ 34\ non\ defective) = P(y = 34) + P(y = 35)$

$$= C_{34}^{35}(0.95)^{34}(0.05)^1 + C_{35}^{35}(0.95)^{35}(0.95)^0$$
$$= 0.3059 + 0.1661$$
$$= 0.4720$$

2 points

iii. $E(x) = np = 35(0.05) = 1.75\ defectives$

$SD(x) = \sqrt{np(1-p)} = \sqrt{35(0.05)(0.95)} = \sqrt{1.6625} = 1.28937$ **2 points**

8) *(9 points)* The scores of 200 students in STAT211 have a normal distribution with mean 70.5 and standard deviation 10,

   i) Find the probability that a student will get a score between 80 and 85.
   ii) How many students will score more than 90?
   iii) The teacher will give A+ to the highest 5%. Find the minimum score to obtain an A+?

Answer:

i. Let X the scores, then X has Normal distribution with mean 7.05 and standard deviation 10, i.e. $X \sim N(7.05, 100)$

$$P(80 < x < 85) = P\left(\frac{80-70.5}{10} < \frac{x-\mu}{\sigma} < \frac{85-70.5}{10}\right)$$
$$= P(0.95 < Z < 1.45) \qquad \textit{where } Z \sim N(0,1)$$
$$= P(0 < Z < 1.45) - P(0 < Z < 0.95)$$
$$= 0.4265 - 0.3289 = 0.0967$$

3 points

ii. $P(X > 90) = P\left(\frac{X-\mu}{\sigma} > \frac{90-70.5}{10}\right)$
$$= P(Z > 1.95) = 0.05 - P(0 < Z < 1.95)$$
$$= 0.5 - 0.4744 = 0.0256$$

Number of the students who will get more than $90 = 0.0256(200) = 5.12$ *students*

4 points

iii. Let the x_0 the maximum score to get A$^+$

$$P(X > x_0) = 0.05$$
$$P\left(\frac{X-\mu}{\sigma} > \frac{x_0-\mu}{10}\right) = 0.05$$
$$P\left(Z > \frac{x_0-\mu}{10}\right) = 0.05$$
$$0.5 - P\left(0 < X < \frac{x_0-\mu}{10}\right) = 0.05$$
$$P\left(0 < X < \frac{x_0-\mu}{10}\right) = 0.45$$
$$\frac{x_0-\mu}{10} = 1.645 \Rightarrow x_0 = 1.645(10) + 70.5 = 86.95$$

2 points