1. 12.5 Examples and hints

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To show that the two lines are smooth, we try to find a point of intersection. We solve the system

$$\begin{aligned} 2+8t_1 &= 5-3t_2 \quad (1)\\ 6-8t_1 &= 5-3t_2 \quad (2)\\ 10t_1 &= 6+t_2 \quad (3) \end{aligned}$$

By solving equations (1), (2) we obtain $\left[t_1 = \frac{1}{4},\ t_2 = \frac{1}{3}\right]$. These values do not satisfy (3). Hence, the two lines are skew.

#32

Three points $P1, P2, P3$ will be on the same line if the two vectors $\overrightarrow{P1P2},\ \overrightarrow{P2P3}$ are parallel. In this problem

$$\overrightarrow{P1P2} = \langle 2, -4, -4 \rangle\,,\ \overrightarrow{P2P3} = \langle 3, -6, -6 \rangle\,.$$

Clearly, $\overrightarrow{P1P2} = \frac{2}{3}\overrightarrow{P2P3}$. Therefore, the two vectors are parallel.

#39, 40

Procedure to find the distance between two parallel lines $L1,\ L2$ (with common direction vector $\mathbf{v}$)

1. Choose two points $P1$ on $L1$ and $P2$ on $L2$ and form the vector $\mathbf{u} = \overrightarrow{P1P2}$.
2. Compute the vectors $\mathbf{w} = \mathrm{Proj}_{\mathbf{v}}\mathbf{u}$ and $\mathbf{z} = \mathbf{u} - \mathbf{w}$.
3. The desired distance is equal $\|\mathbf{z}\|$.

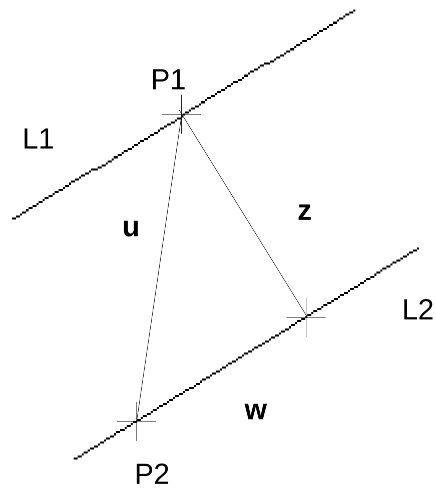

Figure 1:

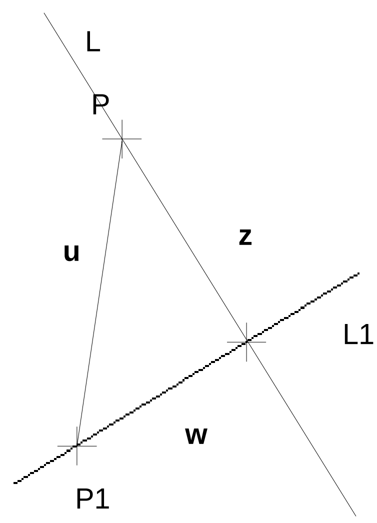

Figure 2:

#47, 48

Procedure to find the parametric equations of a line L that passes through a given point P and is perpindicular ot a given line $L1$ (with direction $\mathbf{v}$).

1. Choose a point $P1$ on $L1$ and form the vector $\mathbf{u} = \overrightarrow{P1P}$.
2. Compute the vectors $\mathbf{w} = \text{Proj}_{\mathbf{v}}\mathbf{u}$ and $\mathbf{z} = \mathbf{u} - \mathbf{w}$.
3. $\mathbf{z}$ is the direction of the desired line.

2. 12.6 Examples and hints

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The direction $\mathbf{v}$ of the line is $\mathbf{v} = \langle 1, 3, 4 \rangle$ and the direction $\mathbf{n}$ of the noral to the plane is $\mathbf{n} = \langle 1, -1, 4 \rangle$. $\mathbf{v} \cdot \mathbf{n} = 14 \neq 0$.Therefore the line is not parallel to the plane. To find where the line intersects the plane substitute from the equations of the line into the equation of the plane. This gives $(1 + t) - (-1 + 3t) + 4\,(2 + 4t) = 0$, Solution is: $t = -\frac{5}{7}$. The point of intersection is $\left(\frac{2}{7}, -\frac{22}{7}, -\frac{6}{7}\right)$.

#35

The plane determined by two parallel lines $L1,\ L2$ (with common direction vector $\mathbf{v}$).

There are several ways to find the equation of such a plane; here is one.

1. Choose two points $P1$ on $L1$ and $P2$ on $L2$ and form the vector $\mathbf{u} = \overrightarrow{P1P2}$.
2. The normal $\mathbf{n}$ to the required plane is $\mathbf{n} = \mathbf{u} \times \mathbf{v}$.
3. Form the equation of the plane using $\mathbf{n}$ and $P1$

#37

To find the equations of the line L of intersection of the two planes $-2x + 3y + 7z + 2 = 0,\ x + 2y - 3z + 5 = 0$:

1. The directio **v** of the line L is the cross producti of the normals to the planes since it is orthogonal to both of them. Therefore,

$$\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & 7 \\ 1 & 2 & -3 \end{vmatrix} = -23\mathbf{i} + \mathbf{j} - 7\mathbf{k}.$$

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2. It remains to find a point P on the line of intersection of the two planes. This is done by assuming a value for one of the three coordinates (say $x = 0$) in the equations of the two planes and then solving the resulting simultanuous system. This gives

$$\begin{aligned} 3y + 7z + 2 &= 0 \\ 2y - 3z + 5 &= 0 \end{aligned}$$

, Solution is: $\left[y = -\frac{41}{23}, z = \frac{11}{23}\right]$. Hence, $P = \left(0, -\frac{41}{23}, \frac{11}{23}\right)$ is on the line L.Using P and **v** we can form the parametric equations of the line of intersection.

Examples 6, 8, 9 in the book should also be consulted to work out other homework problems.