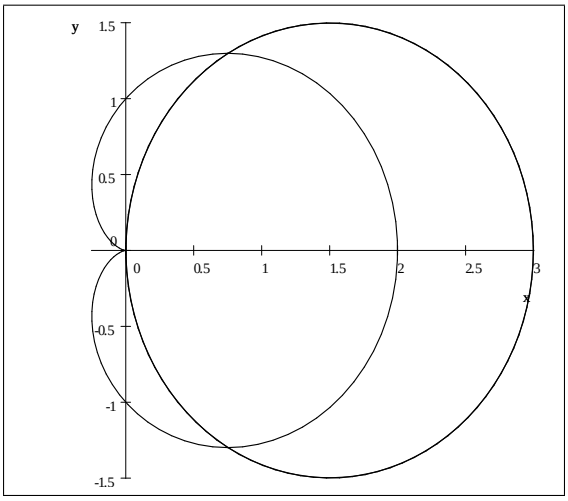
KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
MATH 201-04
Quiz # 2

1. Find all points of intersection of the polar curves $r = 3\cos\theta$ and $r = 1 + \cos\theta$.

   Solution

   The graphs of the two curves are shown below. There is a point of intersection $(0, 0)$ that can be seen from the graph. To get the other two points we solve



$$\begin{aligned} 3\cos\theta &= 1 + \cos\theta \\ \cos\theta &= \frac{1}{2} \\ \theta &= -\frac{\pi}{3}, \frac{\pi}{3} \\ r &= 3\cos\left(\pm\frac{\pi}{3}\right) = \frac{3}{2}. \end{aligned}$$

   The other two points are $\left(\frac{3}{2}, -\frac{\pi}{3}\right)$, $\left(\frac{3}{2}, \frac{\pi}{3}\right)$.

2. Find the equation of the sphere with center at $(1, 2, -4)$ and touches the xy-plane.

   Solution

   Ther radius of the sphere is equal the absolute value of the z-coordinate.

$$r = 4.$$

   The equation of the sphere is

$$(x-1)^2 + (y-2)^2 + (z+4)^2 = 16.$$

3. Find the length of the polar curve $r = \sin^2(\theta/2)$ from $\theta = 0$ to $\theta = \pi$.

Solution

$$\begin{aligned} \frac{dr}{d\theta} &= \sin(\theta/2)\cos(\theta/2) \\ \left(\frac{dr}{d\theta}\right)^2 + r^2 &= \sin^2(\theta/2)\cos^2(\theta/2) + \sin^4(\theta/2) \\ &= \sin^2(\theta/2)\left(\cos^2(\theta/2) + \sin^2(\theta/2)\right) \\ &= \sin^2(\theta/2). \end{aligned}$$

$$\begin{aligned} L &= \int_0^\pi \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta \\ &= \int_0^\pi \sin(\theta/2)\, d\theta = -2\cos(\theta/2)\Big]_0^\pi = 4. \end{aligned}$$