SHOW ALL YOUR WORK

1. Find points on the curve $x = 10 - t^2, y = t^3 - 12$ where the tangent is horizontal or vertical.

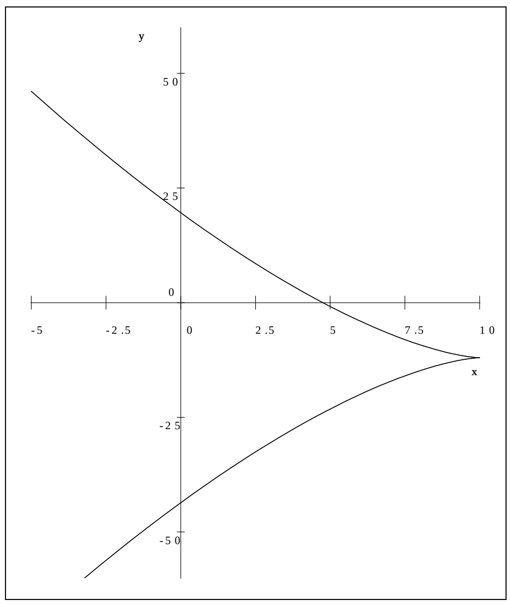
   Solution:

   $$\dot{x} = -2t,\ \dot{y} = 3t^2.$$

   At vertical tangents, $\dot{x} = 0$ and $\dot{y} \neq 0$. The first conditions gives $t = 0$, but $\dot{y} = 0$ at $t = 0$. Therefore there are no vertical tangents.

   At horizontal tangents $\dot{y} = 0$ and $\dot{x} \neq 0$.The first conditions gives $t = 0$, but $\dot{x} = 0$ at $t = 0$. Therefore there are no horizontal tangents.

   The graph of the figure shows also that it does not have any vertical or horizontal tangents.



2. Find$\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the parametric curve $x = 2\sin t,\ y = 3\cos t$.

   Solution:

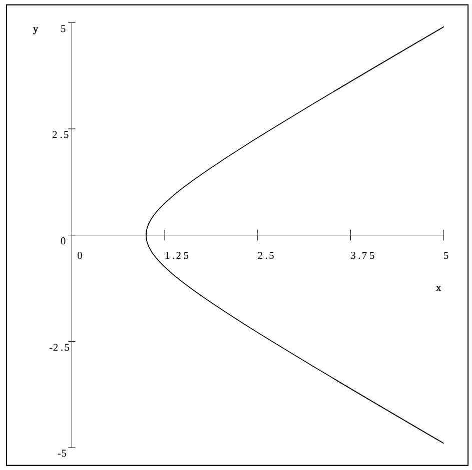
   $$\begin{aligned} \frac{dy}{dx} &= \frac{\dot{y}}{\dot{x}} = -\frac{3\sin t}{2\cos t} = -\frac{3}{2}\tan t., \\ \frac{d^2y}{dx^2} &= \frac{\dot{x}\,\ddot{y} - \dot{y}\,\ddot{x}}{\dot{x}^3} = \frac{-6\cos^2 t - 6\sin^2 t}{8\cos^3 t} = -\frac{3}{4}\sec^3 t. \end{aligned}$$

3. Eleminate t and sketch the resulting curve $x = \sec t, y = \tan t,\ -\frac{\pi}{2} < t < \frac{\pi}{2}$. Indicate with an arrow the direction in which the curve is traced as t increases.

   Solution:

   $$x^2 - y^2 = \sec^2 t - \tan^2 t = 1.$$

   The graph is shown below. It is traced from bottom to top as t increases.


y
5
2.5
0
-2.5
-5
0
1.25
2.5
3.75
5
x