1. Change the polar equation  $r^2 \sin 2\theta = 1$  into rectangular coordinates and sketch the resulting equation.

## Solution:

 $r^2 \cdot 2\sin\theta\cos\theta = 1$ .  $2r\cos\theta r\sin\theta = 1$ . 2xy = 1.

$$y = \frac{1}{2x}.$$



2. Set up an integrarl to compute the area inside the Cardioid  $r = 1 + \cos \theta$  but outside the circle  $r = \sin \theta$ .



## Solution:

The two curves intersect at  $\theta = 0$  and  $\theta = \frac{\pi}{2}$ . The required area is the area inside the cardioid between  $\theta = -\pi$  and  $\theta = \frac{\pi}{2}$  diminished by the area inside the circle between

 $\theta = 0$  and  $\theta = \frac{\pi}{2}$ . Thus

$$A = \int_{-\pi}^{\frac{\pi}{2}} \frac{1}{2} \left(1 + \cos\theta\right)^2 d\theta - \int_{0}^{\frac{\pi}{2}} \frac{1}{2} \sin^2\theta d\theta.$$

3. Find the equation of the sphere whose center is at the point (1, 2, 3) and touches the xy-plane.

Solution:

Since the sphere touches the xy-plane, its radius is the distance between the center and the xy-plane. That is the z-coordinate of the center. Therefore the equation of the sphere is

$$(x-1)^{2} + (y-2)^{2} + (z-3)^{2} = 9.$$