1. Change the polar equation $r^{2} \sin 2 \theta=1$ into rectangular coordinates and sketch the resulting equation.
Solution:
$r^{2} \cdot 2 \sin \theta \cos \theta=1.2 r \cos \theta r \sin \theta=1.2 x y=1$.

$$
y=\frac{1}{2 x} .
$$


2. Set up an integrarl to compute the area inside the Cardioid $r=1+\cos \theta$ but outside the circle $r=\sin \theta$.


Solution:
The two curves intersect at $\theta=0$ and $\theta=\frac{\pi}{2}$. The required area is the area inside the cardioid between $\theta=-\pi$ and $\theta=\frac{\pi}{2}$ diminished by the area inside the circle between
$\theta=0$ and $\theta=\frac{\pi}{2}$. Thus

$$
A=\int_{-\pi}^{\frac{\pi}{2}} \frac{1}{2}(1+\cos \theta)^{2} d \theta-\int_{0}^{\frac{\pi}{2}} \frac{1}{2} \sin ^{2} \theta d \theta
$$

3. Find the equation of the sphere whose center is at the point $(1,2,3)$ and touches the $x y$-plane.
Solution:
Since the sphere touches the $x y$-plane, its radius is the distance between the center and the $x y$-plane. That is the $z$-coordinate of the center. Therefore the equation of the sphere is

$$
(x-1)^{2}+(y-2)^{2}+(z-3)^{2}=9
$$

