Exam # 1 Answers, Oct 2, 2005

1. Give the missing values:

   (a) $\left(-2, -\frac{\pi}{2}\right)$ in polar coordinates $= (2, 0)$ in rectangular coordinates.

   (b) $\left(2, \frac{3\pi}{4}\right)$ in polar coordinates $= \left(-2, \frac{7\pi}{4}\right)$ in polar coordinates.

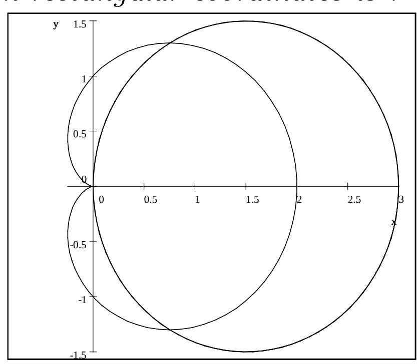
   (c) $\left(-1, \frac{\pi}{3}\right)$ in polar coordinates $= \left(-1\ , -\frac{5\pi}{3}\right)$ in polar coordinates.

   (d) $\left(3, \frac{7\pi}{6}\right)$ in polar coordinates $= \left(-3\ , \frac{\pi}{6}\right)$ in polar coordinates.

   (e) $(2, -2)$ in rectangular coordinates $= \left(-\sqrt{8}, \frac{3\pi}{4}\right)$ in polar coordinates.

2. Find all points of intersection between the cardioid $r = 1 + \cos\theta$ and the circle $\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$, (a) in polar coordinates, and (b) in rectangular coordinates.

   The circle $\left(x - \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$ *in rectangular coordinates is* $r = 3\cos\theta$



   in polar coordinates. Solving with the equation (), we get $1 + \cos\theta = 3\cos\theta$, *or* $\cos\theta = 1/2$. *Therefore,* $\theta = \pm\frac{\pi}{3}$. *Computing, we get* $r = 3/2$. *Thus we have the two points of intersection* $\left(3/2, \frac{\pi}{3}\right)$ *and* $\left(3/2, -\frac{\pi}{3}\right)$. *From the graph, we see that* $(0, 0)$ *is also a point of intesection. In rectangular coordinates, these poins are* $\left(\frac{3}{4}, \frac{3\sqrt{3}}{4}\right)$, $\left(\frac{3}{4}, -\frac{3\sqrt{3}}{4}\right)$ *and* $(0, 0)$.

3. A parametric curve is said to cross itself if it passes through the same point (x, y) for two distinct values of the parameter t. Show that the curve $x = t^3 - 4t,\ y = t^2$ crosses itself at the point $(0, 4)$ and find the equations of the two tangent lines at that point.

   If the curve passes through the point $(0, 4)$, *then* $0 = t^3 - 4t$ *and* $4 = t^2$. *The first equation gives* $t = \pm 2$, $t = 0$ *and the second gives* $t = \pm 2$.*At* $t = 0$, $y = 0$. *Thus,* $t = 0$ *is excluded. The curve crosses itself at* $t = -2$ *and at* $t = 2$. *The slope of the tangent is* $\frac{dy}{dx} = \frac{2t}{3t^2 - 4}$. *At* $t = \pm 2$, $\frac{dy}{dx} = \pm\frac{1}{2}$. *Hence, the equations of the tangent lines are* $y - 4 = \pm\frac{1}{2}x$.

4. Calculate the arclength of the polar curve $r = \sin^3\frac{\theta}{3}$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

$r' = \sin^2\frac{\theta}{3}\cos\frac{\theta}{3}$.

$$\begin{aligned} L &= \int_0^{\frac{\pi}{2}} \sqrt{\sin^6\frac{\theta}{3} + \sin^4\frac{\theta}{3}\cos^2\frac{\theta}{3}}d\theta \\ &= \int_0^{\frac{\pi}{2}} \sin^2\frac{\theta}{3}\sqrt{\sin^2\frac{\theta}{3} + \cos^2\frac{\theta}{3}}d\theta \\ &= \int_0^{\frac{\pi}{2}} \sin^2\frac{\theta}{3}d\theta = \frac{\pi}{4} - \frac{3\sqrt{3}}{8}. \end{aligned}$$

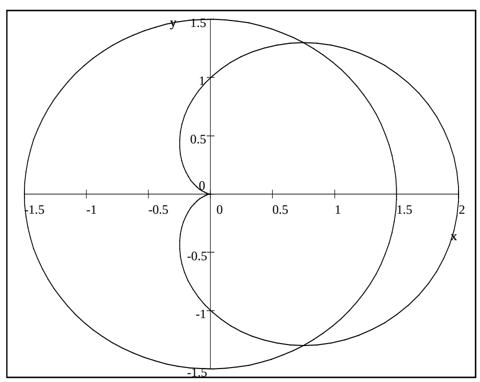
5. Setup an integral to compute the area inside the rose $r = \sin 2\theta$.

   $A = 8\int_0^{\frac{\pi}{4}} \sin^2 2\theta d\theta$.

6. We first find points of intersection of the two curves. Equating the expressions for r we get an integral to compute the area inside the cardioid $r = 1 + \cos\theta$ but outside the circle $r = \frac{3}{2}$.

   We first find points of intersection of the two curves. Equating the expressions for r we get

$$\begin{aligned} \frac{3}{2} &= 1 + \cos\theta \\ \cos\theta &= \frac{1}{2} \\ \theta &= \pm\frac{\pi}{3}. \end{aligned}$$



The area is given by

$$A = 2\int_0^{\frac{\pi}{3}} (1 + \cos\theta)^2 - \frac{9}{4}.$$