1. Change to rectangular coordinates: $(i)(5,2 \pi / 3), \quad(i i)(-5,-\pi / 6)$

Solution 1. (i) $x=5 \cos (2 \pi / 3)=-2.5, y=5 \sin (2 \pi / 3)=2.5 \sqrt{3}$. Therefore, $(x, y)=(-2.5,2.5 \sqrt{3})$
(ii) $x=-5 \cos (-\pi / 6)=-2.5 \sqrt{3}, y=-5 \sin (-\pi / 6)=-2.5$. Therefore, $(x, y)=(-2.5 \sqrt{3}, 2.5)$.
2. Express

1. $x^{2}\left(x^{2}+y^{2}\right)=y^{2}$ as a polar equation and simplify your answer.
2. $\theta=\frac{\pi}{4}$ as a Cartesian equation and simplify your answer.

Solution 2. 1. $x^{2}\left(x^{2}+y^{2}\right)=y^{2} \Longrightarrow r^{2} \cos ^{2} \theta r^{2}=r^{2} \sin ^{2} \theta r^{2} \Longrightarrow r^{2}=$ $\tan ^{2} \theta \Longrightarrow r= \pm \tan \theta$. Since $(r, \theta)$ and $(-r, \theta+\pi)$ are representations of the same point, the answer $r=-\tan \theta$ can be represented as $-r=$ $-\tan (\theta+\pi)=-\tan \theta$. Hence, the answer $r=-\tan \theta$ is the same as $r=\tan \theta$.
2. $\theta=\frac{\pi}{4} \Longrightarrow \tan \theta=1 \Longrightarrow \frac{y}{x}=1 \Longrightarrow y=x$.
3. Find all points of intersection of the line $y=x$ and the cardoid $r=1+\cos \theta$.

Solution 3. The curve $y=x$ has the polar representation $\theta=\frac{\pi}{4}$ or $\theta=\frac{5 \pi}{4}$. Substituting these values in the equation of the second polar curve gives $r=1+\frac{1}{\sqrt{2}}, \quad r=1-\frac{1}{\sqrt{2}}$. Hence, we have the two points of intersection $\left(1+\frac{1}{\sqrt{2}}, \frac{\pi}{4}\right),\left(1-\frac{1}{\sqrt{2}}, \frac{5 \pi}{4}\right)$. From the graphs of the two curves, however, we
get the third point of intersection $(0,0)$.

4. Compute $\frac{d y}{d x}$ and $\frac{d^{2} y}{d x^{2}}$ for $x=\sin t$ and $y=\cos 2 t$ at $t=\pi / 3$.

## Solution 4.

$$
\begin{aligned}
\frac{d y}{d x} & =\frac{d y}{d t} / \frac{d x}{d t}=\frac{-2 \sin 2 t}{\cos t}=\frac{-4 \sin t \cos t}{\cos t}=-4 \sin t \\
\frac{d^{2} y}{d x^{2}} & =\frac{d y^{\prime}}{d t} / \frac{d x}{d t}=\frac{-4 \cos t}{\cos t}=-4
\end{aligned}
$$

At $t=\pi / 3$,

$$
\begin{aligned}
\frac{d y}{d x} & =-4 \frac{\sqrt{3}}{2}=-2 \sqrt{3} \\
\frac{d^{2} y}{d x^{2}} & =-4
\end{aligned}
$$

5. Find the equation of the tangent line to the graph of $r=2 \cos \theta$ at $\theta=\pi / 4$.

## Solution 5.

$$
\begin{aligned}
\frac{d y}{d x} & =\frac{r^{\prime} \sin \theta+r \cos \theta}{r^{\prime} \cos \theta-r \sin \theta} \\
& =\frac{-2 \sin ^{2} \theta+2 \cos ^{2} \theta}{-2 \sin \theta \cos \theta-2 \sin \theta \cos \theta} \\
& =\frac{\cos ^{2} \theta-\sin ^{2} \theta}{-2 \sin \theta \cos \theta}
\end{aligned}
$$

At $\theta=\pi / 4$,

$$
\frac{d y}{d x}=0
$$

Hence, the graph has a horizontal tangent at $\theta=\pi / 4$. The $y$ coordinate corresponding to $\theta=\pi / 4$ is

$$
y=2 \cos (\pi / 4) \sin (\pi / 4)=1
$$

Therefore, the equation of the tangent line is

$$
y=1 .
$$

6. Calculate the length of the polar curve $r=\sin ^{2} \frac{\theta}{2}$ from $\theta=0$ to $\theta=\pi$.

## Solution 6.

$$
\begin{aligned}
L & =\int_{0}^{\pi} \sqrt{r^{\prime 2}+r^{2}} d \theta \\
& =\int_{0}^{\pi} \sqrt{\cos ^{2} \frac{\theta}{2} \sin ^{2} \frac{\theta}{2}+\sin ^{4} \frac{\theta}{2}} d \theta \\
& =\int_{0}^{\pi} \sin \frac{\theta}{2} \sqrt{\cos ^{2} \frac{\theta}{2}+\sin ^{2} \frac{\theta}{2}} d \theta \\
& =\int_{0}^{\pi} \sin \frac{\theta}{2} d \theta=-2\left[\cos \frac{\theta}{2}\right]_{0}^{\pi}=2 .
\end{aligned}
$$

7. Set up an integral to calculate the area common between the two cardoids $r=1+\cos \theta$ and $r=1-\cos \theta$. Do not integrate.

$$
\begin{aligned}
A & =4 \int_{\pi / 2}^{\pi} \frac{1}{2}(1+\cos \theta)^{2} d \theta \\
& =2 \int_{\pi / 2}^{\pi}(1+\cos \theta)^{2} d \theta
\end{aligned}
$$

Solution 8. Find the area of the surface generated by revolving the curve $r=\cos \theta$ about the line $\theta=\pi / 2$.


Figure 1:

## Solution 8.

$$
\begin{aligned}
A & =2 \pi \int_{0}^{\pi} x \sqrt{r^{2}+r^{\prime 2}} d \theta \\
& =2 \pi \int_{0}^{\pi} r \cos \theta \sqrt{\cos ^{2} \theta+\sin ^{2} \theta} d \theta \\
& =2 \pi \int_{0}^{\pi} \cos ^{2} \theta d \theta=\pi^{2}
\end{aligned}
$$

