1. Change to rectangular coordinates: (i) $(5, 2\pi/3)$, (ii) $(-5, -\pi/6)$

Solution 1. (i) $x = 5\cos(2\pi/3) = -2.5$, $y = 5\sin(2\pi/3) = 2.5\sqrt{3}$. Therefore, $(x, y) = (-2.5, 2.5\sqrt{3})$ (ii) $x = -5\cos(-\pi/6) = -2.5\sqrt{3}$, $y = -5\sin(-\pi/6) = -2.5$. Therefore, $(x, y) = (-2.5\sqrt{3}, 2.5)$.

- 2. Express
 - 1. $x^2 (x^2 + y^2) = y^2$ as a polar equation and simplify your answer. 2. $\theta = \frac{\pi}{4}$ as a Cartesian equation and simplify your answer.
 - **Solution 2.** 1. $x^2 (x^2 + y^2) = y^2 \Longrightarrow r^2 \cos^2 \theta \ r^2 = r^2 \sin^2 \theta \ r^2 \Longrightarrow r^2 = \tan^2 \theta \Longrightarrow r = \pm \tan \theta$. Since (r, θ) and $(-r, \theta + \pi)$ are representations of the same point, the answer $r = -\tan \theta$ can be represented as $-r = -\tan(\theta + \pi) = -\tan\theta$. Hence, the answer $r = -\tan\theta$ is the same as $r = \tan \theta$.

2.
$$\theta = \frac{\pi}{4} \Longrightarrow \tan \theta = 1 \Longrightarrow \frac{y}{x} = 1 \Longrightarrow y = x.$$

3. Find all points of intersection of the line y = x and the cardoid $r = 1 + \cos \theta$.

Solution 3. The curve y = x has the polar representation $\theta = \frac{\pi}{4}$ or $\theta = \frac{5\pi}{4}$. Substituting these values in the equation of the second polar curve gives $r = 1 + \frac{1}{\sqrt{2}}$, $r = 1 - \frac{1}{\sqrt{2}}$. Hence, we have the two points of intersection $\left(1 + \frac{1}{\sqrt{2}}, \frac{\pi}{4}\right)$, $\left(1 - \frac{1}{\sqrt{2}}, \frac{5\pi}{4}\right)$. From the graphs of the two curves, however, we





4. Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = \sin t$ and $y = \cos 2t$ at $t = \pi/3$.

Solution 4.

$$\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{-2\sin 2t}{\cos t} = \frac{-4\sin t\cos t}{\cos t} = -4\sin t,$$
$$\frac{d^2y}{dx^2} = \frac{dy'}{dt} / \frac{dx}{dt} = \frac{-4\cos t}{\cos t} = -4.$$

At $t = \pi/3$,

$$\frac{dy}{dx} = -4\frac{\sqrt{3}}{2} = -2\sqrt{3},$$
$$\frac{d^2y}{dx^2} = -4.$$

5. Find the equation of the tangent line to the graph of $r = 2\cos\theta$ at $\theta = \pi/4$.

Solution 5.

$$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$
$$= \frac{-2\sin^2\theta + 2\cos^2\theta}{-2\sin\theta\cos\theta - 2\sin\theta\cos\theta}$$
$$= \frac{\cos^2\theta - \sin^2\theta}{-2\sin\theta\cos\theta}.$$

$$At \ \theta = \pi/4,$$
$$\frac{dy}{dx} = 0.$$

Hence, the graph has a horizontal tangent at $\theta = \pi/4$. The y coordinate corresponding to $\theta = \pi/4$ is

$$y = 2\cos(\pi/4)\sin(\pi/4) = 1.$$

Therefore, the equation of the tangent line is

$$y = 1.$$

6. Calculate the length of the polar curve $r = \sin^2 \frac{\theta}{2}$ from $\theta = 0$ to $\theta = \pi$.

Solution 6.

$$L = \int_0^{\pi} \sqrt{r'^2 + r^2} d\theta$$

=
$$\int_0^{\pi} \sqrt{\cos^2 \frac{\theta}{2} \sin^2 \frac{\theta}{2} + \sin^4 \frac{\theta}{2}} d\theta$$

=
$$\int_0^{\pi} \sin \frac{\theta}{2} \sqrt{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}} d\theta$$

=
$$\int_0^{\pi} \sin \frac{\theta}{2} d\theta = -2 \left[\cos \frac{\theta}{2} \right]_0^{\pi} = 2.$$

7. Set up an integral to calculate the area common between the two cardoids $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$. Do not integrate.

$$A = 4 \int_{\pi/2}^{\pi} \frac{1}{2} (1 + \cos \theta)^2 d\theta$$
$$= 2 \int_{\pi/2}^{\pi} (1 + \cos \theta)^2 d\theta.$$

Solution 8. Find the area of the surface generated by revolving the curve $r = \cos \theta$ about the line $\theta = \pi/2$.



Figure 1:

Solution 8.

$$A = 2\pi \int_0^{\pi} x \sqrt{r^2 + r'^2} d\theta$$
$$= 2\pi \int_0^{\pi} r \cos \theta \sqrt{\cos^2 \theta + \sin^2 \theta} d\theta$$
$$= 2\pi \int_0^{\pi} \cos^2 \theta d\theta = \pi^2.$$