Section 7.5 Strategy for Integration

In this section, we will survey all the techniques we have learned so far. So we shall go through two examples. This section of the text should be easy to read and you should do so.

Example: Evaluate $\int \frac{\sqrt{1+ln(x)}}{xln(x)}dx$

Solution:
Let $u = ln(x) \to du = \frac{dx}{x}$. Then,

$$\int \frac{\sqrt{1+ln(x)}}{xln(x)}dx = \int \frac{\sqrt{1+u}}{u}du$$

Let $1 + u = v^2$, then $du = 2vdv$

$$\int \frac{\sqrt{1+u}}{u}du = \int \frac{v}{v^2-1}2vdv = -\int \frac{2v^2}{1-v^2}dv$$

let $v = sin(p)$, then $dv = cos(p)dp$

$$-\int \frac{2v^2}{1-v^2}dv = -\int \frac{2(sin(p))^2}{(cos(p))^2}cos(p)dp$$

$$-\int \frac{2(sin(p))^2}{cos(p)}dp = -\int \frac{2-2(cos(p))^2}{cos(p)}dp =$$

$$-\int (2cos(p) - 2sec(p))dp = 2sin(p) - 2ln|sec(p) + tan(p)|$$

$$= 2v - 2ln|\frac{1+v}{\sqrt{1-v^2}}| = 2\sqrt{1+ln(x)} + ln|\frac{1+\sqrt{1+ln(x)}}{1-\sqrt{1+ln(x)}}|$$

Example: Evaluate $\int \frac{\sqrt{2x-1}}{2x+3}dx$

Solution:

Let $u^2+4=2x+3$, then $u^2=2x-1$ and $2udu=2dx \to udu=dx$.

$$\int \frac{\sqrt{2x-1}}{2x+3}dx=\int \frac{u^2}{u^2+4}du$$

Choose $u=2\tan\theta \to du=2\sec^2\theta d\theta$.

Then we have: $u^2+4=4\tan^2\theta+4=4\sec^2\theta$

$$\int \frac{u^2}{u^2+4}du=\int \frac{4\tan^2\theta}{4\sec^2\theta}2\sec^2\theta d\theta=\int 2\tan^2\theta d\theta=$$
$$\int 2(\sec^2\theta-1)d\theta=2(\tan\theta-\theta)=\frac{u}{2}-\tan^{-1}(\frac{u}{2})$$

Since $u=\sqrt{2x-1}$, the final answer is $\frac{\sqrt{2x-1}}{2}-\tan^{-1}(\frac{\sqrt{2x-1}}{2})$

Example: Evaluate $\int \frac{e^{2x}}{1+e^x}dx$

Solution:
Let $u = e^x + 1$, then $du = e^x dx$.

$$\int \frac{e^{2x}}{1+e^x}dx = \int \frac{u-1}{u}du$$

$$\int \frac{u-1}{u}du = u - ln(u) = e^x + 1 - ln(e^x + 1)$$