Section 5.6 - The Logarithm defined as an integral

The natural logarithm is defined by

$$ln(x) = \int_1^x \frac{1}{t} dt$$

You should read through this section as it contains a good review of the laws of logarithms which were covered in previous courses.

Example: By comparing areas, show that $\frac{1}{3} < ln(1.5) < \frac{5}{12}$ and use the midpoint rule with $n = 10$ to estimate ln(1.5)

Solution:

We can take ln(1.5) to be the area under the curve $\frac{1}{x}$ between $x = 1$ and $x = 1.5$. Draw the curve $\frac{1}{x}$ for $x \in [1, 1.5]$. To show the inequality, recognize the smallest rectangle (right-end point).

The area of that rectangle is then $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

Next the upper bound of the inequality can be seen in the trapezoid which comprises sides where the left-end and right-end points.

Area of trapezoid $= \frac{1}{2} \times 0.5 \times [1 + \frac{2}{3}] = \frac{5}{12}$

Using midpoint point rule, we have the following sample points x_i: $x_i = 1.025, 1.075,$ $1.125, 1.175, 1.225, 1.275, 1.325, 1.375, 1.425, 1.475.$

Then, $A = \sum_{i=1}^{10} 0.05(\frac{1}{x_i}) = 0.4054$

Now it is review time for 1st quiz!!