Section 11.8 - Power Series A power series is a series of the form: $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \cdots$

More generally, a series of the form: $\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$ is called a power series **centered at a** or **power series about a** or **power series in (x-a)**.

Theorem: For a given power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ there are only three possibilities:

(i) The series converges only when $x = a$.

(ii) The series converges for all x.

(iii) There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$. R is known as the radius of convergence of the power series. The **interval**

of convergence of a power series is the interval that consists of all values of x in which the series converges.

How do we find the radius of convergence?

Example: Find the radius of convergence and the interval of convergence of the series $\sum_{n=1}^{\infty}(-1)^n\frac{(x+2)^n}{n2^n}$.

Let $a_n = (-1)^n\frac{(x+2)^n}{n2^n}$. Convergence occurs when $\lim_{n\to\infty}|\frac{a_{n+1}}{a_n}| = L < 1$.

$$\lim_{n\to\infty}|\frac{a_{n+1}}{a_n}|$$

$$\Rightarrow \lim_{n\to\infty}|\frac{(x+2)^{n+1}}{(n+1)2^{n+1}}\cdot\frac{n2^n}{(x+2)^n}|$$

$$\Rightarrow \lim_{n\to\infty}|\frac{(x+2)}{2}\cdot\frac{n}{n+1}| = |\frac{x+2}{2}| < 1.$$

So the interval of convergence is $-4 < x < 0$. We need to check at the end points, i.e. at $x = 0$ and $x = -4$.

At $x = 0$, the series converges by the alternating series test. At $x = -4$, the resulting series diverges by the p-test. Hence, the interval of convergence is $(-4, 0]$.