Section 11.2 - Series

An infinite series is said to be **convergent** if it all adds up to a particular number. Otherwise, it is **divergent**.

A common type of infinite series is a geometric series which has the form $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$. If $|r| < 1$, then the geometric series is convergent. If $|r| \geq 1$, then the geometric series is divergent.

Theorem: If the infinite series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.
Note that $\lim_{n\to\infty} a_n = 0$ does not necessarily imply that $\sum_{n=1}^{\infty} a_n$ is convergent!!!

Theorem: If $\lim_{n\to\infty} a_n$ does not exist or $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

Example: $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$

Solution:
$\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}} = \sum_{n=1}^{\infty} e(\frac{e}{3})^{n-1}$. Since $e/3 < 1$, the geometric series is convergent and its sum is $\frac{e}{1-e/3}$.

Example: $\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n}$

Solution:
$\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n} = \sum_{n=1}^{\infty} (1/2)(\frac{3}{6})^{n-1} + (1/3)(\frac{2}{6})^{n-1}$

$= (\frac{1/2}{1-1/2}) + \frac{1/3}{1-1/3} = 1 + 1/2 = 3/2.$