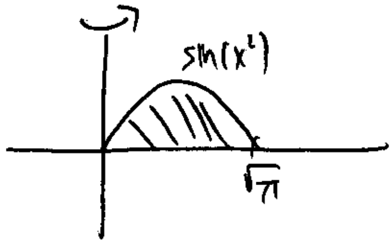
(1)



use shell method:

$$\int_0^{\sqrt{\pi}} 2\pi\, x\, f(x)\,dx$$

$$= \int_0^{\sqrt{\pi}} 2\pi \cdot x \sin(x^2)\,dx.$$

$u = x^2 \Rightarrow du = 2x\,dx$

$x = \sqrt{\pi} \Rightarrow u = \pi$

$x = 0 \Rightarrow u = 0$

$$\int_0^{\pi} \pi \sin(u)\,du = \pi(-\cos u)\Big|_0^{\pi} = (-\cos\pi + \cos 0)\pi$$

$$= 2\pi.$$

Answer is (b).

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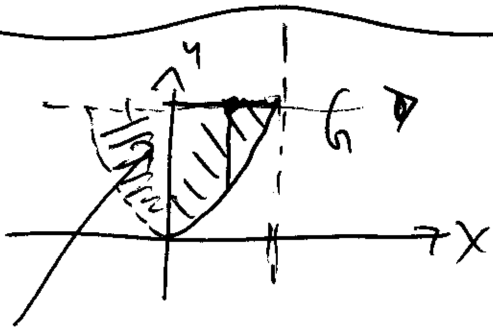
(2)



don't forget this part ∵ the question allows this region as well!!

$r = 1 - x^4$ (volume by disc).

$$V = \int_0^1 \pi (1-x^4)^2 \cdot dx.$$

$$= \int_0^1 \pi (1 - 2x^4 + x^8)\,dx$$

$$= \pi \left[x - \frac{2x^5}{5} + \frac{x^9}{9} \right]_0^1$$

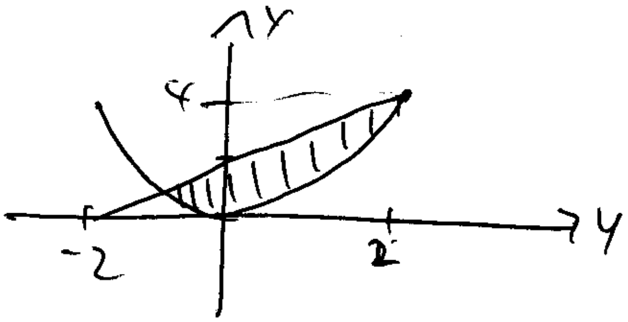
$$= \pi \left(1 - \frac{2}{5} + \frac{1}{9} \right)$$

$$= \pi \left(\frac{45 - 18 + 5}{45} \right) = \frac{32}{45}\pi.$$

$$V_{total} = 2\left(\frac{32\pi}{45}\right) = \frac{64\pi}{45}.$$

Answer is (c).

(3) $x=y^2$ $x=y+2$



$y^2=y+2$

$y^2-y-2=0$

$(y-2)(y+1)=0$

$y=-1$ & $y=2$

$$\int_{-1}^{2}(y+2-y^2)\,dy$$

Answer is (a).

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(4) $$\int \frac{dx}{\sqrt{6x-x^2}} = \int \frac{dx}{\sqrt{9-9+6x-x^2}}$$

$$= \int \frac{dx}{\sqrt{9-(x-3)^2}}$$

let $x-3=3\sin\theta$.

$dx=3\cos\theta\,d\theta$

$$= \int \frac{3\cos\theta\,d\theta}{\sqrt{9-9\sin^2\theta}} = \int \frac{3\cos\theta\,d\theta}{\sqrt{9\cos^2\theta}}$$

$$= \theta$$

$$= \sin^{-1}\left(\frac{x-3}{3}\right)+C.$$

Answer is (a).

(5) $\int_0^3 x f'(1+x^2)\, dx$.

$u = 1+x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int_1^{10} \frac{du}{2} f'(u)$

$= \frac{1}{2} \int_1^{10} f'(u)\, du$

$= \frac{1}{2} [f(10) - f(1)] = 4$

$x=3 \Rightarrow u = 1+9 = 10$

$x=0 \Rightarrow u = 1+0 = 1$

$\Rightarrow f(10) = 8+3 = 11$.

Ans. is (b)

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(6) $\int_0^{\pi^2/4} \cos(\sqrt{x})\, dx$

$u = \sqrt{x} \Rightarrow u^2 = x$

$\Rightarrow 2u\,du = dx$

$x = \frac{\pi^2}{4} \Rightarrow u = \frac{\pi}{2}$

$x = 0 \Rightarrow u = 0$

$\int_0^{\frac{\pi^2}{4}} \cos(\sqrt{x})\, dx = \int_0^{\pi/2} \cos u \, du \cdot 2u$

Integration by pts, $\tilde{u} = 2u$; $d\tilde{v} = \cos u\, du$

$d\tilde{u} = 2du$; $\tilde{v} = \sin u$.

$2u \sin u \Big|_0^{\pi/2} - \int_0^{\frac{\pi}{2}} 2 \sin u\, du$

$\pi \sin(\frac{\pi}{2}) + [2\cos u]_0^{\pi/2} = \pi + 2(0-1)$

$= \pi - 2$

Ans is (c).

(8) $\int_0^1 \frac{1}{1+e^{-x}} dx$

$u-1 = e^{-x}$

$\frac{1}{u-1} = e^{x}$

$u = 1+e^{-x} \Rightarrow du = -e^{-x} dx \Rightarrow dx = -e^{x} du.$

$dx = \frac{-1}{u-1} du.$

$\int_0^1 \frac{1}{1+e^{-x}} dx$

$x=1 \Rightarrow u = 1+e^{-1} = 1+\frac{1}{e}$

$x=0 \Rightarrow u = 1+1 = 2$

$\int \frac{1}{1+e^{-x}} dx = \int \frac{-1}{u-1} \cdot du \cdot \frac{1}{u}$

$= (-1)\int \left(\frac{1}{u(u-1)}\right) du$

$\frac{A}{u} + \frac{B}{u-1} = \frac{1}{u(u-1)}$

$\Rightarrow Au - A + Bu = 1 \Rightarrow -A = 1$

$A = -1$

$A+B=0 \Rightarrow B=1.$

$\int \left(\frac{-1}{u} + \frac{1}{u-1}\right) du = -\ln|u| + \ln|u-1|$

$= -\ln|1+e^{-x}| + \ln|e^{-x}| = \ln\left|\frac{e^{-x}}{1+e^{-x}}\right|$

$-\int_0^1 \frac{1}{1+e^{-x}} dx = \left(+\ln\left|\frac{e^{-1}}{1+e^{-1}}\right| - \ln\left|\frac{1}{2}\right|\right)(-1)$

$= -\ln\left|\frac{1}{e+1}\right| + \ln\left|\frac{1}{2}\right| = \ln\left|\frac{e+1}{2}\right|. \Rightarrow$ Ans is (d).

(9) $\int_4^8 \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

$$\int_4^8 \left(x+2+\frac{1}{x}\right) dx$$

$$= \left(\frac{x^2}{2}+2x+\ln x\right)\Big|_4^8$$

$$= \frac{8^2}{2}+2\cdot 8+\ln 8-\frac{4^2}{2}-2\cdot 4-\ln 4$$

$$= 32+\cancel{16}+3\ln 2-\cancel{8}-8-2\ln 2$$

$$= 32+\ln 2 \Rightarrow \text{Ans is (9)}.$$

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(10) $\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{i}{n}\right)^2} \rightarrow \int_0^1\left(\frac{1}{1+x^2}\right)dx.$

$\Delta x = \frac{1}{n} \qquad x = i\,\Delta x.$

use. $x=\tan\theta \Rightarrow 1+x^2 = 1+\tan^2 = \frac{1}{\cos^2}$

$x = \sec^2\theta\, d\theta \qquad \Rightarrow \frac{1}{1+x^2} = \cos^2$

$$\int_0^1 \frac{1}{1+x^2}dx \rightarrow \int\frac{1}{1+x^2}dx = \int \cos^2\cdot\sec^2\theta\, d\theta = \theta.$$

$$= \tan^{-1}x.$$

$$\int_0^1 \frac{1}{1+x^2}dx = \tan^{-1}(x)\Big|_0^1 = \tan^{-1}(1) = \frac{\pi}{4} \quad \left(\because \tan\left(\frac{\pi}{4}\right)=\frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = 1\right)$$

$\Rightarrow$ Ans is (e)

(12) $\int_0^{\pi/3} \sqrt{1+(y')^2}\, dx$

$y' = \frac{1}{\cos x} \cdot (\sin x)$

$= \int_0^{\pi/3} \sqrt{1 + \frac{\sin^2 x}{\cos^2 x}}\, dx$

$1 + \tan^2 = \frac{1}{\cos^2}$

$= \sec^2$

$\int_0^{\pi/3} \sec x \, dx$

$\frac{\sin(\frac{\pi}{3})}{\cos(\frac{\pi}{3})} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= \sqrt{3}$

$= \int_0^{\pi/3} \frac{\sec x(\sec x + \tan x)}{(\tan x + \sec x)}\, dx$

$\sec(\frac{\pi}{3}) = \frac{1}{\cos(\frac{\pi}{3})} = \underline{2}$

$= \ln|\sec x + \tan x| \Big|_0^{\pi/3}$

$= \ln|\sec(\frac{\pi}{3}) + \tan(\frac{\pi}{3})| - \ln|\sec 0 + \tan 0|$

$= \ln|2+\sqrt{3}| - \ln|1| = \ln|2+\sqrt{3}|$

$\Rightarrow$ Ans is (a).

(13) $\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$

Choose $b_n = \frac{1}{n}$.

$\sum \frac{1}{n-\sqrt{n}} > \sum \frac{1}{n}$ div.

So by comparison test, $\sum \frac{1}{n-\sqrt{n}}$ div.

$\Rightarrow$ Ans is (C).

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(14) $\int \frac{dx}{x^3-x} = \int \frac{dx}{x(x^2-1)} = \int \frac{dx}{x(x-1)(x+1)}$

$\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} = \frac{1}{x(x-1)(x+1)}$ ← partial fractions

$A(x^2-1) + Bx(x+1) + Cx(x-1) = 1$

$Ax^2 - A + Bx^2 + Bx + Cx^2 - Cx = 1$.

$\Rightarrow$ $A = -1$.

$Bx - Cx = 0 \Rightarrow B = C$.

$Ax^2 + Bx^2 + Cx^2 = 0 \Rightarrow A + B + C = 0$.

$-1 + B + B = 0$.

$B = \frac{1}{2}$; $C = \frac{1}{2}$

$$\int \frac{dx}{x^3 - x} = \int \left(\frac{-1}{x} + \frac{1}{2(x-1)} + \frac{1}{2(x+1)} \right) dx$$

$$= -\ln x + \frac{1}{2} \ln|x-1| + \frac{1}{2} \ln|x+1| + C.$$

$$= \frac{1}{2} \ln|x^2 - 1| - \ln x + C.$$

$\Rightarrow$ Ans is (a)

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(15) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}} = \sum_{n=3}^{\infty} \frac{1}{n\sqrt{\ln n}} + \frac{1}{2\sqrt{\ln 2}}$ $\qquad \ln n < n \ (n \geq 3)$

$= \sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}} > \sum_{n=2}^{\infty} \frac{1}{n\sqrt{n}}$ $\qquad \frac{1}{n} < \frac{1}{\ln n}$

div. by p-series..

look at (b) & (e). (b) is rubbish $\because$ test for div. does not work as $\lim_{n\to\infty} \frac{1}{n\sqrt{n}} = 0$.

So left with (e). $\Rightarrow$ Ans (e)

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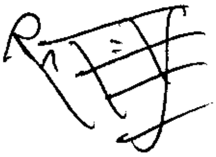
(16) 

$|S_n - S| \leq 10^{-8}$.

$a_{N+1} = \frac{1}{(k+1)^4} \leq 10^{-8}$.

$10^8 \leq (k+1)^4$

$10^2 \leq k+1$

$k \geq 99$

$\Rightarrow$ Ans is (b).

(17) $\int \left(\frac{2}{x+\sqrt[3]{x}}\right) dx$

$u = \sqrt[3]{x}$, $u^3 = x$ $\Rightarrow dx = 3u^2 du.$

$\int \left(\frac{2 \cdot 3u^2}{u^3+u}\right) du$

$\frac{u^2}{u(u^2+1)} = \frac{A}{u} + \frac{Bu+C}{u^2+1}.$

$\Rightarrow Au^2 + A + Bu^2 + Cu = u^2$

$\Rightarrow A+B=1$, $A=0$ $\Rightarrow B=1$. & $C=0$.

$\int \frac{6u}{u^2+1} du$

$v = u^2+1 \Rightarrow \frac{dv}{2} = u\,du$

$\int 6 \cdot \frac{dv}{2} \cdot \frac{1}{v} = 3\ln v = 3 \ln|u^2+1|$

$= 3\ln|x^{\frac{2}{3}}+1|. \Rightarrow$ Ans is (e).

(18) $\int_0^{\pi/2} \cos^2 x \sin 2x \, dx$

$\cos^2 x - \sin^2 x = \cos 2x$

$\cos^2 x - (1 - \cos^2) = \cos 2x$

$u =$

$2\cos^2 x = 1 + \cos 2x$

$\cos^2 x = \dfrac{1 + \cos 2x}{2}$.

$u = \cos 2x + 1$

$\Rightarrow du = (-\sin 2x)(2)\, dx$.

$\dfrac{-du}{2} = \sin 2x \, dx$.

$\int \cos^2 x \sin 2x \, dx = \int -\dfrac{du}{2} \cdot \dfrac{u}{2}$

$= \dfrac{-u^2}{8} \rightarrow \dfrac{-(1+\cos 2x)^2}{8}$

$\int_0^{\frac{\pi}{2}} \cos^2 x \sin 2x \, dx = \dfrac{(1+1)^2}{8} = \dfrac{2^2}{8} = \dfrac{1}{2} \Rightarrow$ Ans is (c)

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(19) $\int \dfrac{1}{x^2(1+x^2)} dx$

$\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{1+x^2} = \dfrac{1}{x^2(1+x^2)}$.

partial fractions. $\Rightarrow Ax(1+x^2) + B(1+x^2) + (Cx+D)x^2 = 1$.

$\Rightarrow Ax + Ax^3 + B + Bx^2 + Cx^3 + Dx^2 = 1$.

$\Rightarrow B = 1$; $B + D = 0$ ~~$\Rightarrow B+D=0$~~ $A + C = 0$

$\Rightarrow D = -1$ $A = 0 \Rightarrow C = 0$

$$\int \frac{dx}{x^2(1+x^2)} = \int \left(\frac{1}{x^2}\right) dx + \int \left(\frac{-1}{x^2+1}\right) dx$$

$\Rightarrow$ clearly (a) & (c) are only answers $\because$ of

$\int \frac{-1}{x^2+1}\,dx$ but (a) is answer $\because$ of

$\int \frac{1}{x^2}\,dx = \frac{-1}{x}$.

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(20) $$\sum_{n=2}^{\infty} \frac{1}{n(n-1)} = ~~\frac{1}{2\cdot 1} + \frac{1}{3\cdot 2} +~~$$

$$\frac{A}{n} + \frac{B}{n-1} = \frac{1}{n(n-1)}$$

$$An - A + Bn = 1$$

$$A + B = 0 \Rightarrow -A = 1 \Rightarrow A = -1 \text{ \& } B = 1$$

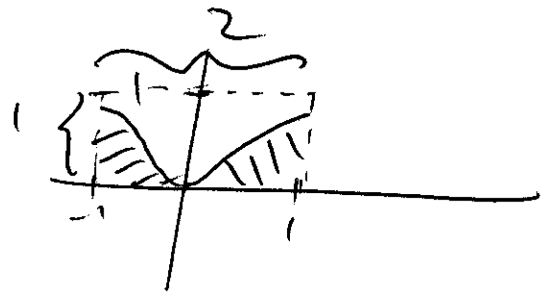
$$\sum_{n=2}^{\infty} \frac{1}{n(n-1)} = \sum_{n=2}^{\infty} \frac{-1}{n} + \frac{1}{n-1}$$

$$= \left(-\frac{1}{2} + \frac{1}{1}\right) - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \cdots$$

$$= 1$$

$\Rightarrow$ Ans is (e).

(21) $I = \int_{-1}^{1} \sin(x^2)\,dx = 2\int_0^1 \sin(x^2)$



$\sin(x^2) = 1$ @ $x^2 = \frac{\pi}{2}$

$x = \frac{\sqrt{\pi}}{\sqrt{2}} > 1.$

Area of $\sin(x^2)$ can't be more than rectangle shown.

So. (b) → not even an improper integral, so not (b)

(c) → positive area, i.e. area is abv x-axis, not (c)

(e) → ∵ of (c), not (e).

(d) → area less than square as shown in Fig.
So look @ both side, $I > 2$ is impossible

⇒ Ans. is (a)

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(22) $\int xe^{-x}\,dx$

$u = x$ $\quad dv = e^{-x}dx$
$du = dx$ $\quad v = -e^{-x}$

$-xe^{-x}\Big|_0^{\infty} + \int_0^{\infty} e^{-x}\,dx = +1$ ⇒ Ans is (e)

$-e^{-x}\Big|_0^{\infty} = -(-1)$

(24) $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$.

$x=2 \Rightarrow e^2 \Rightarrow$ (b)

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(25) $\lim_{k\to\infty}\left|\frac{(-1)^{k+1}\,(k+1)!}{e^{k+1}}\cdot\frac{e^k}{(-1)^k\,k!}\right|$

$= \lim_{k\to\infty}\left|\frac{k+1}{e}\right| = \infty \Rightarrow$ div. $\Rightarrow$ (d)

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(26) AST: $\lim_{n\to\infty}\frac{1}{n^4\sqrt{n}} = 0$, $\frac{1}{n^4\sqrt{n}}\downarrow$ $\Rightarrow$ conv. by AST.

$n^4\cdot n^{1/2} = n^{9/2}$ $\quad \frac{9}{2}>1 \Rightarrow$ conv by p-series
$\Rightarrow$ abs conv. $\Rightarrow$ (a)

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(27) $f'(x) = \ln x + 1$ $\qquad f^{(4)}(x) = \frac{2}{x^3}$ $\qquad \sum C_n(x-1)^n$

$f''(x) = \frac{1}{x}$ $\qquad f^{(5)}(x) = \frac{-2\cdot 3}{x^4}$ $\qquad C_n = \frac{f^{(n)}(1)}{n!}$

$f'''(x) = \frac{-1}{x^2}$

@ $x=1$, coeff $C_2>0$, (d) eliminated

$C_3<0$, (b) & (c) eliminated

$C_5 = -\frac{1}{20} \Rightarrow$ (a) only.

(28) $\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{(n+1)^3 2^{n+1}} \cdot \frac{(n)^3 \cdot 2^n}{(x-2)^n} \right| < 1.$

$= \lim_{n\to\infty} \left| \frac{(x-2)}{2} \right| < 1.$

$\Rightarrow |x-2| < 2.$

$\Rightarrow 0 < x < 4.$

At $x=0 \Rightarrow \sum \frac{(-2)^n}{n^3 2^n} = \sum \frac{(-1)^n}{n^3}$

$\Rightarrow$ conv. by AST.

$x=4 \Rightarrow \sum \frac{2^n}{n^3 2^n} \Rightarrow$ conv by p-series

$\therefore$ $0 \leq x \leq 4$

$\Rightarrow$ (c)

(23)

$$\ln(1-x) = \int \frac{-1}{(1-x)}\,dx.$$

$$-\ln(1-x) = \int \left(\frac{1}{1-x}\right) dx$$

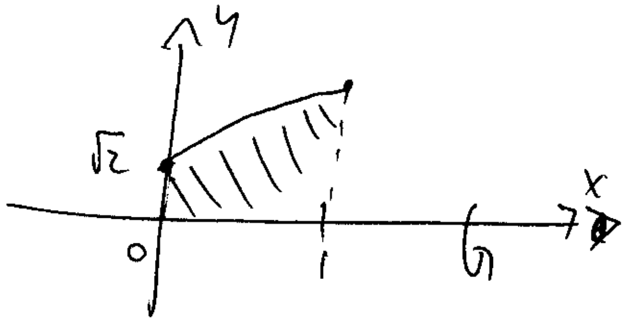
$$\ln\left(\frac{1}{1-x}\right) = \int \left(\frac{1}{1-x}\right) dx = \int \left(\sum_{n=0}^{\infty} x^n\right) dx$$

$$\ln\left(\frac{1}{1-x}\right). = \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)} = \sum_{k=1}^{\infty} \frac{x^n}{n}.$$

set $k = n+1$.

@ $x = 1/3$, $\ln\left(\frac{1}{1-\frac{1}{3}}\right) = \ln\left|\frac{1}{\left(\frac{2}{3}\right)}\right| = \ln\left|\frac{3}{2}\right| \Rightarrow$ (c)

(11)



$$SA = \int 2\pi f(x)\, ds$$

$$= \int_0^1 2\pi \sqrt{1+e^x} \cdot (1+(y')^2)^{1/2} dx$$

$$y = \sqrt{1+e^x} \Rightarrow y' = \frac{1}{2}(1+e^x)^{-1/2} \cdot e^x$$

$$= \frac{e^x}{2\sqrt{1+e^x}}$$

$$(y')^2 = \frac{e^{2x}}{4(1+e^x)} \qquad \sqrt{1+(y')^2} = \sqrt{\frac{4(1+e^x)+e^{2x}}{4(1+e^x)}}$$

$$SA = \int_0^1 \pi \left(\sqrt{e^{2x}+4e^x+4}\right) dx$$

$$= \int_0^1 \pi \sqrt{(e^x+2)^2}\, dx$$

$$= \int_0^1 \pi(e^x+2)\, dx = \pi e^x + 2\pi x \Big|_0^1$$

$$= \pi e + 2\pi - \pi$$

$$= \pi e + \pi$$

$$= \pi(e+1)$$

$\Rightarrow$ Ans is (a)