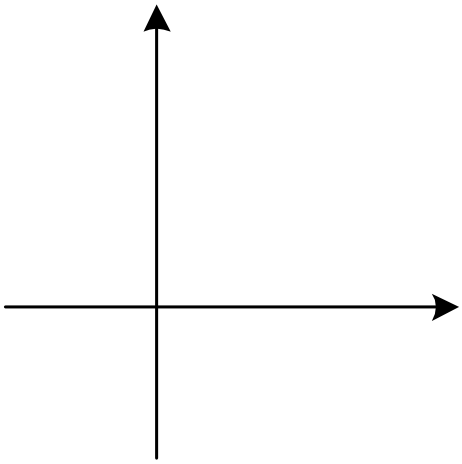
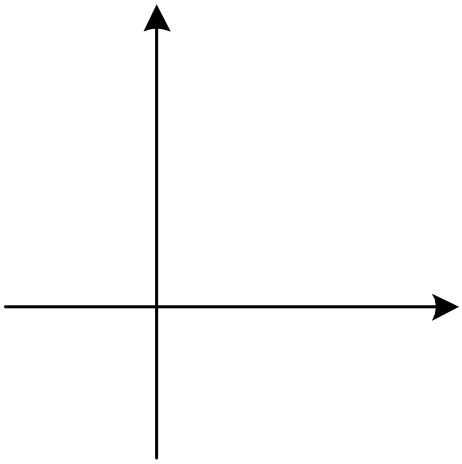
1. **[6 + 6 points]** Using the method of Cylindrical Shells, **set up**, but **do not evaluate**, an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = e^{2x}$, $y = e^2$, and $x = 0$

   (a) about the line $x = -2$ [Sketch the region and a typical rectangle]



   (b) about the x-axis. [Sketch the region and a typical rectangle]



2. **[7 points]** Find the average value of the function $f(x) = x\sec^2(2x)$ on the interval $\left[0, \frac{\pi}{8}\right]$.

3. Evaluate each of the following integrals:

(a) **[6 points]** $\int \sin^5(3t)\cos^4(3t)\,dt.$

(b) **[6 points]** $\int \frac{\sqrt{x}}{x+4}\,dx.$

(c) **[8 points]** $\int \frac{1}{1+2\sin x}\,dx.$ Hint: Use the substitution $t = \tan\left(\frac{x}{2}\right)$.

(d) **[7 points]** $\displaystyle\int_{2/5}^{4/5} \frac{\sqrt{25x^2-4}}{x}\,dx.$

(e) **[9 points]** $\displaystyle\int \frac{4x}{(x+1)(x-1)^2}\,dx.$

4. (a) Determine if the integral is improper or not. Justify.

   (i) **[3 points]** $\displaystyle\int_2^5 \ln(x-1)\,dx.$

   (ii) **[3 points]** $\displaystyle\int_1^5 \frac{1}{x^2-x}\,dx.$

   (b) **[6 points]** Determine if the integral $\displaystyle\int_{-\infty}^0 \frac{1}{\sqrt{3-x}}\,dx$ is convergent or divergent. If it is convergent, find its value.

5. **[6 points]** Determine whether the sequence $\left\{\dfrac{\sin(3n)}{3n+1}\right\}_{n=1}^{+\infty}$ converges or diverges. If it converges, find its limit.

6. Consider the series $\displaystyle\sum_{n=1}^{+\infty} \frac{\ln(n+2)-\ln(n+1)}{\ln(n+1)\cdot\ln(n+2)}$.

   (a) **[5 points]** Find a formula for S_n, the sequence of partial sums.

   (b) **[4 points]** Determine whether the given series converges or diverges. If it converges, find its sum.

7. Determine whether the series converges or diverges. Justify your answer.

(a) **[5 points]** $\displaystyle\sum_{n=1}^{+\infty} \sin^{-1}\left(\frac{n}{n+1}\right)$.

(b) **[5 points]** $\displaystyle\sum_{n=0}^{+\infty} \frac{(2e)^{n+1}}{3^{2n-1}}$.

(c) **[8 points]** $\displaystyle\sum_{n=1}^{+\infty} n\, e^{-2n}$.