King Fahd University of Petroleum and Minerals
Deanship of Scientific Research

Internal Research Grant Proposal "MS/DOMAIN/369"

"`Subalgebras of Affine Domains over a Noetherian Domain`"

$\rightarrow$ duration = 18 months $\leftarrow$

Prof. S. Kabbaj, KFUPM
(External Investigator: Prof. S. Bouchiba, Univ. of Meknes)

Contents

1 Abstract

In this project, we are concerned with the Krull dimension theory of subalgebras of affine domains over Noetherian integral domains. In particular, we investigate Anderson-Dobbs-Eakin-Heinzer's problem on whether the Krull setting occurring in the generalized fourteenth problem of Hilbert yields (locally) Jaffard domains. Our strategy consists of framing our work in the larger context of Bouvier conjecture on finite-dimensional non-Noetherian Krull domains.

MATHEMATICS SUBJECT CLASSIFICATION: Primary 13C15, 13F05, 13F15; Secondary 13E05, 13F20, 13G05, 13B25, 13B30.

KEYWORDS: Noetherian domain, Krull domain, factorial domain, Finitely generated algebra, Krull dimension, valuative dimension, Jaffard domain, fourteenth problem of Hilbert.

2 Introduction and Literature Review

All rings considered are commutative with identity. A finite-dimensional domain R is said to be Jaffard if $\dim(R[X_1, ..., X_n]) = n + \dim(R)$ for all $n \geq 1$; equivalently, if $\dim(R) = \dim_v(R)$, where $\dim(R)$ denotes the (Krull) dimension of R and $\dim_v(R)$ its valuative dimension (i.e., the supremum of dimensions of the valuation overrings of R). As this notion does not carry over to localizations, R is said to be locally Jaffard if R_p is a Jaffard domain for each prime ideal p of R (equiv., $S^{-1}R$ is a Jaffard domain for each multiplicative subset S of R). The class of locally Jaffard domains contains most of the well-known classes of rings involved in Krull dimension theory such as Noetherian domains, Prüfer domains, universally catenarian domains, and universally strong S-domains. We assume familiarity with the above concepts, as in [3, 5, 11, 12, 17, 25, 27, 29]. Any unreferenced material is standard, as in [22, 28, 30].

In this research project, we are concerned with dimension theory of subalgebras of affine domains over Noetherian integral domains. In particular, we investigate Anderson-Dobbs-Eakin-Heinzer's open problem on whether the Krull setting occurring in the generalized fourteenth problem of Hilbert yields (locally) Jaffard domains. To this purpose, we'll frame our work in the larger context of Bouvier conjecture which sustains the existence of finite-dimensional non-Jaffard Krull (or factorial) domains [12].

Finite-dimensional non-Noetherian Krull (or factorial) domains are scarce in the literature and one needs to test them and their localizations as well for the Jaffard property. One of these family stems from the generalized fourteenth problem of Hilbert (also called Zariski-Hilbert problem): Let k be a field of characteristic zero and let T be a normal affine domain over k. Let F be a subfield of the field of fractions of T. Set $R := F \cap T$. The Hilbert-Zariski problem asks whether R is an affine domain over k. Counterexamples on this problem were constructed by Rees [34], Nagata [32] and Roberts [35, 36]. In this line, Anderson, Dobbs, Eakin, and Heinzer [4] asked whether R and its localizations inherit from T the Noetherian-like main behavior of having Krull and valuative dimensions coincide.

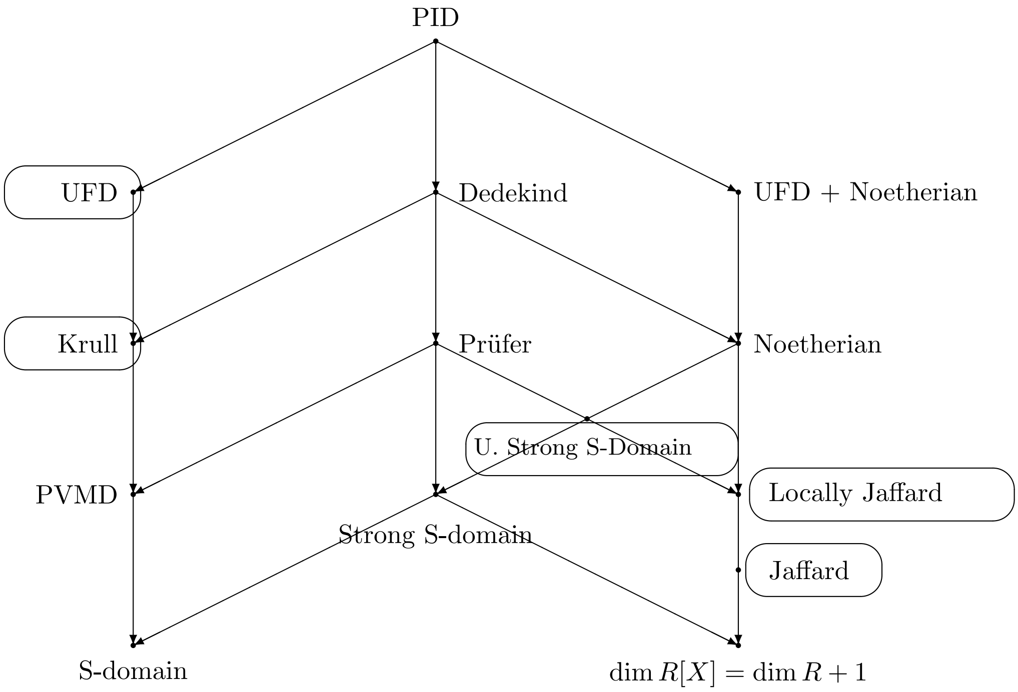
This problem will be addressed within the more general context of subalgebras of affine domains over Noetherian domains; namely, let $A \subseteq R$ be an extension of domains where A is Noetherian and R is a subalgebra of an affine domain T over A. Our ultimate goal is to examine the possible transfer of the Jaffard and locally Jaffard properties to R and hence compute its Krull and valuative dimensions. We'll draw on Ayache-Cahen's work on valuative radical and subintersections [8].

We'll very likely have to restrict our study to the class of Krull rings which stands as a host context for all open problems mentioned above. However, the literature suffers from a lack of systematic study of the prime ideal structure of Krull rings and polynomial rings over Krull rings. One has then to explore this aspect of this class of domains in an attempt to parallel analogous developments on Noetherian rings and polynomial rings over Noetherian rings.

3 Objectives and Brief Description

3.1 Finite-dimensional non-Noetherian Krull domains

It is an open problem to compute the dimension of polynomial rings over Krull domains in general. In this vein, Bouvier conjectured that "finite-dimensional Krull domains need not be Jaffard" [12, 20]. The following diagram places this conjecture in its proper perspective. In particular, it indicates how the classes of finite dimensional Noetherian domains, Prüfer domains, UFDs, Krull domains, and PVMDs [22] interact with the notion of (locally) Jaffard domain as well as with the (strong) S-domain properties of Kaplansky [27, 28, 29].



Obviously, Bouvier conjecture makes sense beyond the Noetherian context. As the notion of Krull domain is stable under formation of rings of fractions and adjunction of indeterminates, it merely claims the existence of a Krull domain R and a multiplicative subset S such that $1 + \dim(S^{-1}R) \lneqq \dim(S^{-1}R[X])$. However, finite-dimensional non-Noetherian Krull domains are scarce in the literature and one needs to test them and their localizations as well for the Jaffard property. Our main objective here is to study all these families of examples as to whether they are subject or not to the (locally) Jaffard property.

3.2 Subalgebras issued from the Zariski-Hilbert problem

Let k be a field of characteristic zero and let T be a normal affine domain over k. Let F be a subfield of the field of fractions of T. Set $R := F \cap T$. The Hilbert-Zariski problem asks whether R is an affine domain over k. Counterexamples on this problem were constructed by Rees, Nagata and Roberts. In this line, Anderson, Dobbs, Eakin, and Heinzer [4, p. 214] asked whether R and its localizations inherit from T the Noetherian-like main behavior of having Krull and valuative dimensions coincide (i.e., R is locally Jaffard). This question is addressed, in this section, within a more general context; namely, subalgebras of affine domains over Noetherian domains.

Let $A \subseteq R$ be an extension of domains where A is Noetherian. Assume R is a subalgebra of an affine domain T over A. Here we aim to investigate necessary and sufficient conditions for R to inherit the Jaffard and locally Jaffard properties. In this vein, we plan an intensive investigation of the prime ideal structure of subalgebras of affine domains over a Noetherian domain. For this purpose, we will place our focus on those domains R lying between a Noetherian domain A and its localization $A[a^{-1}]$ for some nonzero element a of A. The motivation behind this reduction lies in the fact -we have been able to prove so far- that "R is a subalgebra of an affine domain over A if and only if there exists $a \neq 0 \in R$ such that $R[a^{-1}]$ is an affine domain over A if and only if there exist an affine domain B over A and $a \neq 0 \in B$ such that $B \subseteq R \subseteq B[a^{-1}]$.

3.3 Case of symbolic Rees algebra over Noetherian domains

Here we will focus on the special case of symbolic Rees algebras. In 1958, Rees constructed in [34] a first counter-example to the Zariski-Hilbert problem (initially posed at the Second International Congress of Mathematicians at Paris in 1900). His construction gave rise to (what is now called) Rees algebras. Since then, these special graded algebras have been capturing the interest of many mathematicians, particularly in the fields of commutative algebra and algebraic geometry. Let A be an integral domain, t an indeterminate over A, and $p \in \mathrm{Spec}(A)$. For each $n \in \mathbb{Z}$, set $p^{(n)} := p^n A_p \cap A$, the n^{th} symbolic power of p, with $p^{(n)} = A$ for each $n \leq 0$. We recall the following definitions:

- $\bigoplus_{n\in\mathbb{Z}} p^n t^n = A[t^{-1}, pt, ..., p^n t^n, ...]$ is the Rees algebra of p.
- $\bigoplus_{n\in\mathbb{Z}} p^{(n)} t^n = A[t^{-1}, p^{(1)}t, ..., p^{(n)}t^n, ...]$ is the symbolic Rees algebra of p.

In 1970, based on Rees' work, Eakin and Heinzer constructed in [18] a first example of a 3-dimensional non-Noetherian Krull domain. It arose as a symbolic Rees algebra. This enhances our interest for these constructions. As an application of our work in the previous section, we wish to push further the analysis of the prime deal structure of symbolic Rees algebras. Particularly, we plan to investigates the lifting of prime ideals of $A[t^{-1}]$ containing t^{-1} in the symbolic Rees algebra R. Further, we'll study the length of chains of prime ideals of R lying over a common prime ideal of $A[t^{-1}]$.

4 Scheduling of the Proposed Research

Any precise scheduling could be highly speculative. Nonetheless, we hope to be able to prove some non-trivial results for each of the above tasks through the following schedule:

Task 1:	"Finite-dimensional non-Noetherian Krull domains"	**6 months**
Task 2:	"Subalgebras issued from the Zariski-Hilbert problem"	**6 months**
Task 3:	"Case of symbolic Rees algebra over Noetherian domains"	**6 months**

5 Personnel Requirements

(None)

6 Monitoring and Evaluation

In Pure Mathematics, I can think of no better way to evaluate the project findings than to have them published in refereed journals of good reputation.

7 Budget

1.	Faculty 18-month compensation for S. Kabbaj	21,600 SR
2.	1-month visit of Prof. S. Bouchiba (Ext. Co-I) to KFUPM	7,000 SR
3.	Stationary	500 SR
	TOTAL =	**29,100** SR

8 Resume

8.1 Bio

- `Degrees`
  - 1993: Habilitation in Mathematics, Univ. of Lyon 1, Lyon, France
  - 1988: Ph.D. in Mathematics, Univ. of Lyon 1, Lyon, France
  - 1985: MS in Mathematics, Univ. of Lyon 1, Lyon, France
  - 1983: Maitrise in Mathematics, Univ. of Poitiers, Poitiers, France
  - 1982: Licence in Mathematics, Univ. of Poitiers, Poitiers, France

- `Employment History`
  - 1998 - Professor (1998-), KFUPM, Dhahran, Saudi Arabia
  - 1994 - Professor, University of Fez, Fez, Morocco
  - 1989-93 Associate Professor, University of Fez, Fez, Morocco
  - 1986-87 Assistant Associ, University of Lyon 1, Lyon, France

- `Visiting Positions`
  - 2001: Harvard University, Cambridge, USA (One year)
  - 1997: University of Tennessee, Knoxville, USA (Spring term)
  - 1988-96: Univ. of Roma, Rome, Italy (Several visits)
  - 1993: University of Virginia, Charlottesville, USA (Summer term)

- `Short Research Visits (with presentations)`
  - University of Tennessee, Knoxville, USA (1987)
  - University of Aix-Marseille III, France
  - University of Barcelona, Barcelona, Spain
  - University of Sfax, Sfax, Tunisia
  - University of North Carolina, Charlotte, USA (1997)
  - IPM, School of Mathematics, Tehran, Iran (2004)

- `Research Interest`
  - Class groups and class semigroups
  - Cores and Lipman adjoints of ideals in non-Noetherian settings
  - Dual of an ideal and trace properties
  - Gröbner bases and computational methods
  - Homological aspects of commutative rings
  - Ideal systems, star operations, and semi-star operations
  - Integer-valued polynomial rings
  - Prime spectra and dimension theory
  - Pullbacks
  - Tilting theory in commutative settings
  - Trivial ring extensions

- `Graduate Supervision`
  - 13 Doctoral dissertations + several MS theses at Fez, Morocco
  - 2 Ph.D. (in progress) + 2 MS theses (1 in progress) at KFUPM, KSA

- `Teaching Activities`
  - two Courses per semester (all levels)
  - E-learning and partial online delivery via WebCT (Math 131 & 102)
  - Course coordination (lower division courses)
  - Curriculum development (all levels)
  - Development of graduate programs (KFUPM & Univ. of Fez)
  - Lecture series (for graduate students & junior faculty)
- `Grants & Awards`
  - Distinguished Graduate Fellowship (1985-87, France)
  - CNR Grants (1992, 1994, 1996, Italy)
  - HCM European Commission Research Award (1993)
  - Fulbright Grant (1993, USA)
  - ICTP Regular Associate (1997-2002, Italy)
  - Shoman Prize in Mathematics (1998, Pan-Arab)
  - AFESD Distinguished Scholar Award (2001, Pan-Arab)
  - Distinguished Researcher in the College of Sciences (2004, KFUPM)
  - Al-Marai Distinguished Scientist Prize (2005, Saudi Arabia)
- `Languages`
  - Arabic
  - English
  - French
  - Italian

8.2 Research Papers

- In all papers, names of authors appeared in alphabetical order
- The total number of non-self citations amounts to 163

[1] La formule de la dimension pour les S-domaines forts universels, *Bollettino dell' Unione Matematica Italiana Algebra e Geometria*, vol V-D (1) (**1986**) 145–161. [Cited 11 times]

[2] Examples of Jaffard domains (with Bouvier), *J. Pure Appl. Algebra* 54 (**1988**) 155–165. [Cited 6 times]

[3] On Jaffard domains (with Anderson, Bouvier, Dobbs & Fontana), *Expo. Math.* 6 (**1988**) 145–175. [Cited 34 times]

[4] Universally catenarian domains of D+M type (with Anderson, Dobbs and Mulay), *Proc. Amer. Math. Soc.* 104 (2) (**1988**) 378–384. [Cited 6 times]

[5] Sur une conjecture de Malik et Mott, *Pub. Math. Lyon I series* 3/B (**1988**) 13–19.

[6] Sur l'anneau factoriel non noethérien de Fujita, *Proceedings of the 1988 Rome Conference, Univ. of Roma I*, (**1989**) 125–130.

[7] On the Krull and valuative dimension of $D + XDS[X]$domains (with Fontana), *J. Pure Appl. Algebra* 63 (**1990**) 231–245. [Cited 8 times]

[8] Une conjecture sur les anneaux de Nagata, *J. Pure Appl. Algebra* 64 (**1990**) 263–268. [Cited 5 times]

[9] Direct limits of Jaffard domains and S-domains (with Dobbs & Fontana), *Comment. Math. Univ. St. Paul.* 39 (2) (**1990**) 143–155. [Cited 3 times]

[10] Sur les S-domaines forts de Kaplansky, *J. Algebra* 137 (2) (**1991**) 400–415. [Cited 12 times]

[11] Krull and valuative dimension of the rings of the form$A + XB[X]$ (with Fontana & Izelgue), *Dekker Lect. Notes Pure Appl. Math.* 153 (**1993**) 111–130. [Cited 5 times]

[12] Quelques propriétés des chaines d'idéaux dans les anneaux $A + XB[X]$ (with Fontana & Izelgue), *Communication in Algebra* 22 (1) (**1994**) 9–27. [Cited 12 times]

[13] Duals of ideals in pullback constructions (with Houston,Lucas & Mimouni), *Dekker Lect. Notes Pure Appl. Math.* 171 (**1995**) 263–276. [Cited 2 times]

[14] Dimension of the tensor product of two particular pullbacks (with Girolami), *Kluwer Math. Appl.* 343 (**1995**) 221– 226.

[15] Sur quelques propriétés des sous-anneaux de la forme $D + I$ d'un anneau intègre (with Fontana & Izelgue), *Communications in Algebra* 23 (11) (**1995**) 4189–4210. `[Cited 6 times]`

[16] Classes of $D + M$ rings defined by homological conditions(with Costa), *Comm. Algebra* 24 (3) (**1996**) 891–906. `[Cited 1 time]`

[17] Group rings $R[G]$ with 3-generated ideals when R is Artinian (with Ameziane & Fontana), *Comm. Algebra* 24 (4) (**1996**) 1253–1280. `[Cited 4 times]`

[18] Dimension and valuative dimension of the Serre conjecturer ing $R < X >$ (with Cahen & Elkhayyari), *Dekker Lect. Notes Pure Appl. Math.* 185 (**1997**) 173–180. `[Cited 4 times]`

[19] Dimension of tensor products of AF-rings (with Bouchiba & Girolami), *Dekker Lect. Notes Pure Appl. Math.* 185 (**1997**) 141–154.

[20] Duals of ideals in polynomial rings (with Houston, Lucas & Mimouni), *Dekker Lect. Notes Pure Appl. Math.* 185 (**1997**) 349–358. `[Cited 1 time]`

[21] Some factorization properties of $A + XB[X]$ domains (with Barucci & Izelgue), *Dekker Lect. Notes Pure Appl. Math.* 185 (**1997**) 69–78. `[Cited 16 times]`

[22] $n-$Coherent rings and modules (with Dobbs & Mahdou), *Dekker Lect. Notes Pure Appl. Math.* 185 (**1997**) 269–282.

[23] Group rings $R[G]$ with 4-generated ideals when R is an Artinian principal ideal ring (with Ameziane), *Dekker Lect. Notes Pure Appl. Math.* 185 (**1997**) 1–14. `[Cited 4 times]`

[24] On the Krull dimension of domains on integer-valued polynomials (with Fontana, Izelgue & Tartarone), *Expo. Math.* 15 (**1997**) 433–465.

[25] On the prime spectrum of commutative semi-group rings (with Ameziane & Dobbs), *Comm. Algebra* 26 (8) (**1998**) 2559–2589. `[Cited 2 times]`

[26] On the spectrum of a group ring (with Ameziane, Costa & Zarzuela), *Comm. Algebra* 27 (1) (**1999**) 387–403. `[Cited 1 time]`

[27] Group rings $R[G]$ with 4-generated ideals when R is an Artinian ring with the two generator property (with Ameziane), *Dekker Lect. Notes Pure Appl. Math.* 205 (**1999**) 1–23. `[Cited 1 time]`

[28] The dimension of tensor products of commutative algebras over a zero-dimensional ring (with Bouchiba & Girolami), *Dekker Lect. Notes Pure*

Appl. Math. 205 (**1999**) 145–159.

[**29**] Trace properties and integral domains (with Lucas & Mimouni), *Dekker Lect. Notes Pure Appl. Math.* 205 (**1999**) 421–436. `[Cited 1 time]`

[**30**] On the class group of $A+XB[X]$ domains (with Anderson & El Baghdadi), *Dekker Lect. Notes Pure Appl. Math.* 205 (**1999**) 73–85. `[Cited 3 times]`

[**31**] When is $D+M$ n-coherent and an $(n,d)-$domain? (with Dobbs, Mahdou & Sobrani), *Dekker Lect. Notes Pure Appl. Math.* 205 (**1999**) 257–270.

[**32**] Polynomial closure in essential domains and pullbacks (with Fontana, Izelgue & Tartarone), *Dekker Lect. Notes Pure Appl. Math.* 205 (**1999**) 307-321. `[Cited 1 time]`

[**33**] The dimension of tensor products of $k-$algebras arising from pullbacks (with Bouchiba & Girolami), *J. Pure Appl. Algebra* 137 (2) (**1999**) 125–138. `[Cited 1 time]`

[**34**] When is the dual of an ideal a ring? (with Houston, Lucas & Mimouni), *J. Algebra* 225 (1) (**2000**) 429–450. `[Cited 2 times]`

[**35**] The Dilworth number of group rings over an Artin local ring(with Ameziane, Okon, & Vicknair), *Comm. Algebra* 28 (10) **(2000**) 4596–4610.

[**36**] On the class group of a graded domain (with El Baghdadi & Izelgue), *J. Pure Appl. Algebra* 171 (2-3) (**2002**) 171–184. `[Cited 4 times]`

[**37**] The homogeneous class group of $A+XB[X]$ domains (with Anderson & El Baghdadi) *Int. J. Comm. Rings* 1 (1) **(2002)** 11–25.

[**38**] Tensor products of Cohen-Macaulay rings. Solution to a problem of Grothendieck (with Bouchiba), *J. Algebra* 252 **(2002)** 65–73. `[Cited 3 times]`

[**39**] On the prime ideal structure of tensor products of algebras (with Bouchiba & Dobbs), *J. Pure Appl. Algebra* 176 **(2002)** 89–112. `[Cited 1 time]`

[**40**] Trivial extensions of local rings and a conjecture of Costa (with Mahdou), *Dekker Lect. Notes Pure Appl. Math.* 231 **(2003)** 301–311.

[**41**] Trace properties and pullbacks (with Lucas & Mimouni), *Comm. Algebra* 31 (3) **(2003)** 1085-1111. `[Cited 1 time]`

[**42**] Class semigroups of integral domains (with Mimouni), *J. Algebra* 264 **(2003)** 620–640.

[**43**] Trivial extensions defined by coherent-like conditions (with Mahdou), *Comm. Algebra* 32 (10) **(2004)** 3937–3953. `[Cited 1 time]`

[44] Essential domains and two conjectures in dimension theory (with Fontana), *Proc. Amer. Math. Soc.* 132 **(2004)**, 2529–2535. `[Cited 2 time]`

[45] On the dimension theory of polynomial rings over pullbacks, in "Multiplicative Ideal Theory in Commutative Algebra," Springer, pp. 263–277, **2006**.

[46] t-Class semigroups of integral domains (with Mimouni), *J. Reine Angew. Math.*, to appear.

8.3 Books and Volumes Edited

• BOOKS EDITED

[1] Commutative Ring Theory (with P.-J. Cahen, D. Costa and M. Fontana), Lecture Notes in Pure and Applied Mathematics, Vol. **153**, Marcel Dekker, INC., New York, **1994**.
(**280 pages**) (ISBN: 0-8247-9170-3)

[2] Commutative Ring Theory II (with P.-J. Cahen, M. Fontana and E. Houston), Lecture Notes in Pure and Applied Mathematics, Vol. **185**, Marcel Dekker, INC., New York, **1997**.
(**488 pages**) (ISBN: 0-8247-9815-5)

[3] Advances in Commutative Ring Theory (with D. Dobbs and M. Fontana), Lecture Notes in Pure and Applied Mathematics, Vol. **205**, Marcel Dekker, INC., New York, **1999**.
(**576 pages**) (ISBN: 0-8247-7147-8)

[4] Commutative Ring Theory and Applications (with M. Fontana and S. Wiegand), Lecture Notes in Pure and Applied Mathematics, Vol. **231**, Marcel Dekker, INC., New York, **2003**.
(**494pages**) (ISBN: 0-8247-0855-5)

• VOLUMES EDITED

[1] Commutative Algebra (with M. Fontana and E. Houston), Arab.J. Sci. Eng. Section C Theme Issue 26, KFUPM, Dhahran, **2001**. (**198 pages**)

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[1] S. Abhyankar, *Two notes on formal power series*, Proc. Amer. Math. Soc. **7** (1956) 903–905.

[2] S. Ameziane, D. Costa, S. Kabbaj, and S. Zarzuela, *On the spectrum of the group ring*, Comm. Algebra **27** (1) (1999) 387–403.

[3] D. F. Anderson, A. Bouvier, D. E. Dobbs, M. Fontana, and S. Kabbaj, *On Jaffard domains*, Exposition. Math. **6** (2) (1988) 145–175.

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[9] M. Atiyah and I. Macdonald, *Introduction to commutative algebra*, Addison-Wesley, Reading, MA, 1969.

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[13] J. W. Brewer, D. L. Costa, and E. L. Lady, *Prime ideals and localization in commutative group rings*, J. Algebra **34** (1975) 300–308.

[14] P.-J. Cahen and A. Ayache, *Radical valuatif et sous-extensions*, Comm. Algebra **26** (9) (1998) 2767–2787.

[15] J. David, *A non-Noetherian factorial ring*, Trans. Amer. Math. Soc. **169** (1972) 495–502.

[16] J. David, *A characteristic zero non-Noetherian factorial ring of dimension three*, Trans. Amer. Math. Soc. **180** (1973) 315–325.

[17] D. E. Dobbs, M. Fontana, and S. Kabbaj, *Direct limits of Jaffard domains and S-domains*, Comment. Math. Univ. St. Paul. **39** (2) (1990) 143–155.

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[20] M. Fontana and S. Kabbaj, *Essential domains and two conjectures in dimension theory*, Proc. Amer. Math. Soc. **132** (2004) 2529–2535.

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[23] R. Gilmer, *A two-dimensional non-Noetherian factorial ring*, Proc. Amer. Math. Soc. **44** (1974) 25–30.

[24] J. M. Giral, *Krull dimension, transcendence degree and subalgebras of finitely generated algebras*, Arch. Math. (Basel) **36** (4) (1981) 305–312.

[25] P. Jaffard, *Théorie de la dimension dans les anneaux de polynômes*, Mém. Sc. Math. 146, Gauthier-Villars, Paris, 1960.

[26] S. Kabbaj, La formule de la dimension pour les S-domaines forts universels, Boll. Un. Mat. Ital. D (6) **5** (1) (1986) 145–161.

[27] S. Kabbaj, *Sur les S-domaines forts de Kaplansky*, J. Algebra **137** (2) (1991) 400–415.

[28] I. Kaplansky, *Commutative rings*, The University of Chicago Press, Chicago, 1974.

[29] S. Malik and J. L. Mott, *Strong S-domains*, J. Pure Appl. Algebra **28** (3) (1983) 249–264.

[30] H. Matsumura, *Commutative ring theory*, Second edition, Cambridge Studies in Advanced Mathematics, 8. Cambridge University Press, Cambridge, 1989.

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[32] M. Nagata, *On the fourteenth problem of Hilbert*, in "Proceedings of the International Congress of Mathematicians, 1958," pp. 459–462, Cambridge University Press, London-New York, 1960.

[33] M. Nagata, *Local rings*, Robert E. Krieger Publishing Co., Huntington, N.Y., 1975.

[34] D. Rees, *On a problem of Zariski*, Illinois J. Math. **2** (1958) 145–149.

[35] P. Roberts, *A prime ideal in a polynomial ring whose symbolic blow-up is not Noetherian*, Proc. Amer. Math. Soc. **94** (1985) 589–592.

[36] P. Roberts, *An infinitely generated symbolic blow-up in a power series ring and a new counterexample to Hilbert's fourteenth problem*, J. Algebra **132** (1990) 461–473

[37] P. Samuel, *Anneaux gradués factoriels et modules reflexifs*, Bull. Soc. Math. France **92** (1964) 237-249

[38] A. R. Wadsworth, *Hilbert subalgebras of finitely generated algebras*, J. Algebra **43** (1) (1976) 298–304.

[39] A. R. Wadsworth, *Hilbert subalgebras of finitely generated algebras II*, Comm. Algebra **5** (8) (1977) 863–872.

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