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Driss BENNIS

Dimensions Homologiques de Gorenstein

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Prof. S. Bouchiba	Univ. de Meknes	Morocco	Membre Rapporteur
Prof. A. Bouvier	Haut Conseil de l'Educ.	France	Président
Prof. F. Couchot	Univ. de Caen	France	Membre Rapporteur
Prof. M. Fontana	Università di Roma Tre	Italy	Membre
Prof. S. Glaz	Univ. of Connecticut	USA	Membre
Prof. S. Kabbaj	KFUPM	KSA	Membre
Prof. N. Mahdou	FST Univ. de Fez	Morocco	Directeur de Thèse
Prof. M. Sobrani	FST Univ. de Fez	Morocco	Membre Rapporteur
Prof. S. Yassemi	IPM-Tehran	Iran	Membre Rapporteur

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Author's papers involved in this thesis:

1. D. Bennis and N. Mahdou, *Strongly Gorenstein projective, injective, and flat modules*, J. Pure Appl. Algebra **210** (2007), 437-445.
2. D. Bennis, N. Mahdou, and K. Ouarghi, *Rings over which all modules are strongly Gorenstein projective*. Accepted for publication in Rocky Mountain Journal of Mathematics.
3. D. Bennis, *Rings over which the class of Gorenstein flat modules is closed under extensions*. Accepted for publication in Communications in Algebra.
4. D. Bennis and N. Mahdou; *Global Gorenstein dimensions of polynomial rings and of direct products of rings*. Accepted for publication in Houston Journal of Mathematics.
5. D. Bennis and N. Mahdou, *Global Gorenstein Dimensions and Cotorsion Dimension of Rings*. Accepted for publication in Communications in Algebra.
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Other papers:

1. D. Bennis, *Note on Gorenstein Flat Dimension*. Submitted for publication.
2. D. Bennis and N. Mahdou, *On n -Perfect Rings and Cotorsion Dimension*. Submitted for publication.
3. D. Bennis, S. E. Kabbaj, and N. Mahdou; *First, second, and third change of rings theorems for Gorenstein homological dimensions*. In preparation.
4. D. Bennis and N. Mahdou, *n -Perfectness in pullbacks*. In preparation.

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