1. (3-points) Find the equation of the line that is parallel to the line $2x - y = -5$ and passes through the point $(2, -1)$.

The slope m of the required line =

The slope of $2x - y = -5$ --- 1 pt

$\Rightarrow$ $m = 2$ --- 1 pt

$\Rightarrow$ The required equation is

$y + 1 = 2(x - 2)$ --- 1 pt

2. (3-points) Let $f(x) = 5x^2 - 6x - 4$. Find all values of x for which $f(x) = -5$.

$5x^2 - 6x - 4 = -5$

$\Rightarrow$ $5x^2 - 6x + 1 = 0$ --- 1 pt

$\Rightarrow$ $(5x - 1)(x - 1) = 0$ --- 1 pt

$\Rightarrow$ $x = \frac{1}{5}$ or $x = 1$ --- 1 pt

3. (3-points) Determine whether the graph of $y = -\sqrt{9-x^2}$ has symmetry with respect to either the x-axis, the y-axis, or the origin.

Symmetry about:

x-axis: $y \to -y \Rightarrow -y = -\sqrt{9-x^2}$

$\Rightarrow y = \sqrt{9-x^2}$ No ... 1pt

y-axis: $x \to -x \Rightarrow y = -\sqrt{9-(-x)^2}$

$= -\sqrt{9-x^2}$ yes .. 1pt

the-origin: $x \to -x$ and $y \to -y \Rightarrow$

$-y = -\sqrt{9-(-x^2)}$

$\Rightarrow y = \sqrt{9-x^2}$ No ... 1pt

4. (3-points) Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ of the function $f(x) = -5x^2 + 4$.

$\frac{1}{h}\left[f(x+h) - f(x)\right] = \frac{1}{h}\left[-5(x+h)^2 + 4 - (-5x^2+4)\right]$ ... 1pt

$= \frac{1}{h}\left[-5x^2 - 10xh - 5h^2 + 4 + 5x^2 - 4\right]$

$= \frac{1}{h}\left[-10xh - 5h^2\right]$ ... 1pt

$= -10x - 5h$ ... 1pt

5. (3-points) Use long division to find the quotient $Q(x)$ when $3x^4 - 5x^3 + 3x - 1$ is divided by $x + 2$.

$$
\begin{array}{r|l}
 & 3x^3 - 11x^2 + 22x - 41 \\
\hline
x+2 & 3x^4 - 5x^3 \qquad + 3x - 1 \\
 & 3x^4 + 6x^3 \\
\hline
 & -11x^3 \qquad 3x - 1 \\
 & -11x^3 - 22x^2 \\
\hline
 & 22x^2 + 3x - 1 \\
 & 22x^2 + 44x \\
\hline
 & -41x - 1 \\
 & -41x - 82 \\
\hline
 & 81
\end{array}
$$

... 1pt

... 1pt

$$\Rightarrow Q(x) = 3x^3 - 11x^2 + 22x - 41 \quad \text{... 1pt}$$

6. (3-points) Find the domain of the function $f(x) = \sqrt{18 - x^2}$. Write your answer in interval notation.

We must have $18 - x^2 \geq 0$ ... 1pt

$\Rightarrow |x| \leq \sqrt{18}$ ... 1pt

$\Rightarrow -3\sqrt{2} \leq x \leq 3\sqrt{2}$

i.e. Domain $= [-3\sqrt{2}, 3\sqrt{2}]$

7. (4-points) Find the vertex and the equation of the axis of symmetry of the graph of $f(x) = -2x^2 + 12x - 16$.

The coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

where $a = -2$, $b = 12$ --- 1 pt

$\Rightarrow -\frac{b}{2a} = -\frac{12}{-4} = 3$

and $f\left(-\frac{b}{2a}\right) = f(3) = -18 + 36 - 16 = 2$ ... 1 pt

$\Rightarrow$ Vertex is at $(3, 2)$ --- 1 pt

The equation of the axis of symmetry

$x = -\frac{b}{2a}$

$\Rightarrow x = 3$ --- 1 pt

8. (4-points) Find the value of the constant k for which the lines $(k+1)x - 4y + 10 = 0$ and $y = \frac{5}{k}x + 11$ are perpendicular.

The slope of $(k+1)x - 4y + 10 = 0$ is $m_1 = \frac{k+1}{4}$ ... 1 pt

The slope of $y = \frac{5}{k} + 11$ is $m_2 = \frac{5}{k}$ --- 1 pt

$\Rightarrow m_1 m_2 = -1$ i.e $\frac{5(k+1)}{4k} = -1$ --- 1 pt

$\Rightarrow 5k + 5 = -4k \Rightarrow 9k = -5$

$\Rightarrow k = -\frac{5}{9}$ --- 1 pt

9. (3-points) Find the equation of the graph which is obtained from $y = x^3$ by means of a reflection across the x-axis, a horizontal shift 2 units to left, and a vertical shift 7 units up.

Reflection across the x-axis $\Rightarrow$ $y = -x^3$

--- 1 pt

Horizontal shift 2 units to left $\Rightarrow$

$y = -(x+2)^3$ --- 1 pt

Vertical shift 7 units up $\Rightarrow$

$y = -(x+2)^3 + 7$ --- 1 pt

10. (3-points) Let $P(x) = 2x^3 + 3x^2 + 8x - 8$. Use the remainder theorem to find $P(-2i)$, where $i = \sqrt{-1}$.

$$\begin{array}{r|rrrr} -2i & 2 & 3 & 8 & -8 \\ & & -4i & -8-6i & -12 \\ \hline & 2 & 3-4i & -6i & -20 \end{array}$$

--- 2 pts

$\Rightarrow$ $P(-2i)$ = The remainder $= -20$ --- 1 pt

11. (3-points) Let $[\![y]\!]$ be the greatest integer less than or equal to y. If $g(x) = [\![x+1]\!] - 2$, find the exact value of each of the following

(a) $g(-\pi)$.

$$-4 < -\pi < -3$$

$$\Rightarrow \quad -3 < -\pi + 1 < -2 \quad \cdots \quad \tfrac{1}{2}\ pt$$

$$\Rightarrow \quad g(-\pi) = [\![-\pi+1]\!] - 2$$

$$= -3 - 2 = -5 \quad \cdots \quad 1\ pt$$

(b) $3g\left(\frac{11}{3}\right) = 3\left[[\![\frac{11}{3}+1]\!] - 2 \right]$

$$= 3\left[[\![\frac{14}{3}]\!] - 2 \right] \quad \cdots \quad \tfrac{1}{2}\ pt$$

But $4 < \frac{14}{3} < 5 \quad \cdots \quad \tfrac{1}{2}\ pt$

$$\Rightarrow \quad 3g\left(\tfrac{11}{3}\right) = 3\left[(4) - 2\right] = 6 \quad \cdots \quad \tfrac{1}{2}\ pt$$

12. (4-points) Let $f(x) = \dfrac{6}{x-2}$ and $g(x) = \dfrac{3}{5x}$. Find each of the following

(a) $(f \circ g)(x)$. $= f(g(x)) \quad \cdots \quad \tfrac{1}{2}\ pt$

$$= \frac{6}{\frac{3}{5x} - 2} \quad \cdots \quad 1\ pt$$

$$= \frac{30x}{3 - 10x} \quad \cdots \quad \tfrac{1}{2}\ pt$$

(b) $(g \circ f)(5)$. $= g(f(5)) \quad \cdots \quad \tfrac{1}{2}\ pt$

$$= g\left(\tfrac{6}{3}\right) = g(2) \quad \cdots \quad 1\ pt$$

$$= \frac{3}{10} \quad \cdots \quad \tfrac{1}{2}\ pt$$

13. (3-points) Find the coordinates of the point of intersection of the lines $3x - y + 7 = 0$ and $2x + y + 8 = 0$.

$\Rightarrow$ $y = 3x + 7$ and $y = -2x - 8$

$\Rightarrow$ $3x + 7 = -2x - 8$ ... 1 pt

$\Rightarrow$ $5x = -15 \Rightarrow x = -3$ ... 1 pt

$\Rightarrow y = 3x + 7 = -2$

$\Rightarrow$ The required point is $(-3, -2)$ ... 1 pt

14. (4-points) Let f be a function such that $f(-3) = 4$. Give the coordinates of a point on the graph of each of the following functions:

(a) $y = f(x - 2)$. Horizontal shift 2 units to right of $(-3, 4) \Rightarrow \boxed{(-1, 4)}$ ... 1 pt

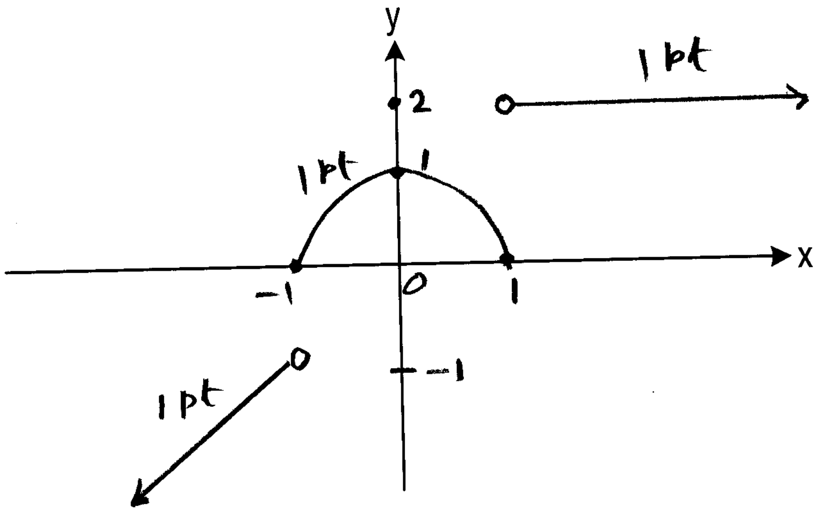
(b) $y = f(x) + 3$. Vertical shift 3 units up to $(-3, 4) \Rightarrow \boxed{(-3, 7)}$ ... 1 pt

(c) $y = f(-x)$. Reflection across the y-axis to $(-3, 4) \Rightarrow \boxed{(3, 4)}$ ... 1 pt

(d) $y = -f(x)$. Reflection across the x-axis to $(-3, 4) \Rightarrow \boxed{(-3, -4)}$ ... 1 pt.

15. (4-points) Let $h(x) = \begin{cases} x & , \quad x < -1 \\ 1 - x^2 & , \quad -1 \le x \le 1 \\ 2 & , \quad x > 1 . \end{cases}$

(a) Sketch the graph of h.



(b) Find the interval on which $\boxed{h \text{ is decreasing.}}$

$[0, 1]$ 1 pt

16. (4-points) Use synthetic division to verify that $x - 3$ is a factor of $P(x) = x^4 - 3x^3 + 2x^2 - 7x + 3$, and write $P(x)$ as the product of $x - 3$ and the reduced polynomial $Q(x)$.

$$\left.\begin{array}{r|rrrrr} 3 & 1 & -3 & 2 & -7 & 3 \\ & & 3 & 0 & 6 & -3 \\ \hline & 1 & 0 & 2 & -1 & 0 \end{array}\right\} \text{---2 pts}$$

$\Rightarrow$ The remainder $= 0$
$\Rightarrow$ $x - 3$ is a factor of $P(x)$ } ...1 pt

$\Rightarrow P(x) = (x-3)(x^3 + 2x - 1)$ --- 1 pt.

17. (3-points) Find two numbers whose difference is 24 and whose product is minimum.

Possible numbers are x and $x-24$

--- 1 pt

$\Rightarrow$ The Product $f(x) = x(x-24)$

$= x^2 - 24x$ --- 1 pt

minimum at $x = -\frac{b}{2a} = -\frac{-24}{2} = 12$

$\Rightarrow$ The Numbers are 12 and -12

1 pt

18. (3-points) Let $f(x) = \sqrt{3-4x}$ and $g(x) = 4x^2 - 8$, find the domain of $\frac{f}{g}$.

$\frac{f}{g}(x) = \frac{\sqrt{3-4x}}{4x^2-8}$

where $3 - 4x \geq 0$ and $x \neq \pm\sqrt{2}$

--- 1 pt

$\Rightarrow$ $x \leq \frac{3}{4}$ and $x \neq \pm\sqrt{2}$ --- 1 pt

i.e Domain $= \{x \mid x \leq \frac{3}{4}$ and $x \neq -\sqrt{2}\}$

--- 1 pt.