

**King Fahd University of Petroleum & Minerals**  
**Department of Mathematics and Statistics**  
**MATH 101 – Calculus I**  
**EXAM I**  
**2010-2011 (103)**

**Tuesday, July 12, 2011**

**Allowed Time: 2 Hours**

**Name:** \_\_\_\_\_

**ID Number:** \_\_\_\_\_ **Serial Number:** \_\_\_\_\_

**Section Number:** \_\_\_\_\_ **Instructor's Name:** \_\_\_\_\_

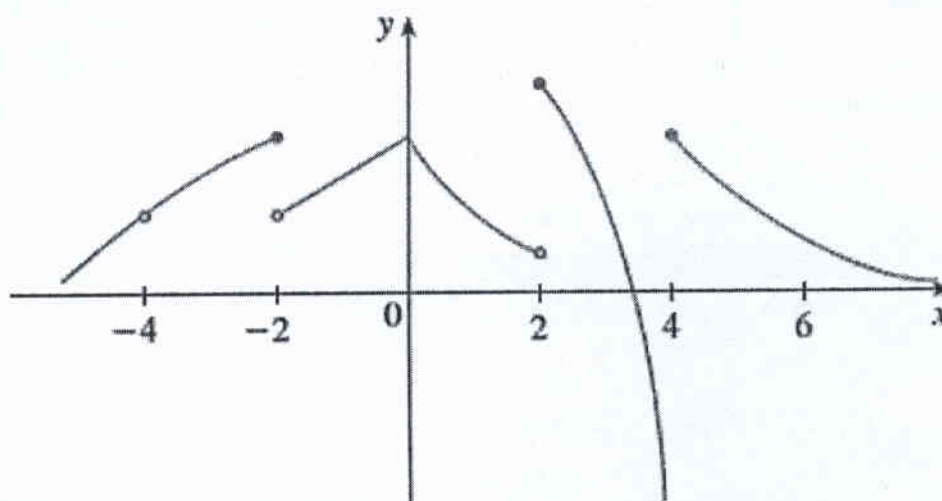
**Instructions:**

1. Write neatly and legibly. You may lose points for messy work.
2. **Show all your work.** No points for answers without justification.
3. **Calculators and Mobiles are not allowed.**
4. Make sure that you have 13 different problems (9 pages + cover page).

<b>Problem No.</b>	<b>Points</b>	<b>Maximum Points</b>
<b>1</b>		<b>12</b>
<b>2</b>		<b>16</b>
<b>3</b>		<b>6</b>
<b>4</b>		<b>4</b>
<b>5</b>		<b>12</b>
<b>6</b>		<b>7</b>
<b>7</b>		<b>5</b>
<b>8</b>		<b>6</b>
<b>9</b>		<b>7</b>
<b>10</b>		<b>8</b>
<b>11</b>		<b>7</b>
<b>12</b>		<b>4</b>
<b>13</b>		<b>6</b>
<b>Total:</b>		<b>100</b>

1. (a) (8 points) From the graph of  $f$ , state the numbers at which  $f$  is discontinuous and explain why.

- (b) (4 points) For each of the numbers stated in part (a), determine whether  $f$  is continuous from the right, or from the left, or neither.



2. Evaluate the limit, if it exists:

(a) (3 points)  $\lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{x^2 + x} \right).$

(b) (3 points)  $\lim_{x \rightarrow 0^-} \frac{|x| + \lfloor [x] \rfloor}{x}.$

(c) (4 points)  $\lim_{x \rightarrow 0} \sin^2 x \cdot \cos \left( \frac{e^x}{x} \right).$

(d) (3 points)  $\lim_{x \rightarrow 0} \frac{\sin^3 x}{\cos^2 x - 1}.$

(e) (3 points)  $\lim_{x \rightarrow \frac{\pi}{2}^+} e^{\tan x} + \ln \left( x - \frac{\pi}{2} \right).$

3. (6 points) Find numbers  $a$  and  $b$  such that

$$\lim_{x \rightarrow 1} \frac{\sqrt{a x + b} - 2}{x - 1} = 2.$$

4. (4 points) Determine whether the function

$$f(x) = \frac{\sqrt{2x+1} - \sqrt{x+1}}{x}$$

has a removable discontinuity, a jump discontinuity, or an infinite discontinuity at  $x = 0$ .

5. (12 points) Use limits to find all vertical and horizontal asymptotes of the graph of

$$y = \frac{x - 3}{\sqrt{x^2 - 4}}.$$

(Justify your answer)

8. (6 points) Find the values of  $A$  and  $B$  that make  $f$  continuous everywhere

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & , \quad \text{if } x < 2 \\ A x^2 - B x + 3 & , \quad \text{if } 2 \leq x < 3 \\ 2 x - A + B & , \quad \text{if } x \geq 3 \end{cases}$$

9. (7 points) Use the Intermediate Value Theorem to show that the equation  $x^3 - x - 1 = 0$  has at least one real root.

10. A particle moves on a line away from its initial position so that after  $t$  hours it is  $s(t) = 3t^2 + t$  miles from its initial position.

(a) (3 points) Find the average velocity of the particle over the interval  $[1, 3]$ .

(b) (5 points) Find the instantaneous velocity at  $t = 1$ .

11. (7 points) Determine whether  $f'(0)$  exists where

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & , \quad \text{if } x \neq 0 \\ 0 & , \quad \text{if } x = 0 \end{cases}$$

(Justify your answer)

12. (4 points) The following limit represents the derivative of some function  $f$  at some number  $a$ . State such an  $f$  and  $a$ . (Give a reason to your answer)

$$\lim_{t \rightarrow 1} \frac{t^4 + t - 2}{t - 1}.$$



13. (6 points) Using the definition of the tangent line as the limit of secant lines, find the equation of the tangent line of the function  $f(x)$  at the point  $(3, 5)$  where  $\lim_{x \rightarrow 3} \frac{f(x) - 5}{x - 3} = 2$ .