Quiz .... Math 101

ID:	Name:	

Q1: Find the critical numbers of the function $f(x)=x^{\frac{4}{5}}(x-4)^2$.

Domain $\mathbb{R}$

$$f'(x)=\frac{4}{5}x^{-\frac{1}{5}}(x-4)^2+x^{\frac{4}{5}}\,2(x-4)$$

$$=\frac{(14x^2-64x+64)}{5\sqrt[5]{x}}$$

c.p. $x=0$ or

$$x=\frac{64\pm\sqrt{64^2-4(14)64}}{28}$$

3 c.p.

Q2: Find the absolute maximum and absolute minimum values of $f(x)=x^3-6x^2+5$ on the interval [-3 , 5]

$f'(x)=3x(x-4)$

x	$f(x)$	
-3	-76	← Abs. min
0	5	← Abs max
4	-27	
5	-20	

Q3: Verify that the function $f(x)=\frac{1}{x}$ **satisfies** the hypotheses of the Mean Value Theorem on the interval [1 , 3]. **And find** all numbers that satisfy the conclusion of the Mean Value Theorem.

$f(x)$ cont $[1,3]$ & diff $(1,3)$ only not diff $x=0$

$\Rightarrow$ it satisfies MVT in $[1,3]$

find c $f'(x)=\frac{-1}{x^2}$ $f'(c)=-\frac{1}{c^2}$

$$-\frac{1}{c^2}=\frac{f(3)-f(1)}{3-1}=\frac{\frac{1}{3}-1}{2}=\frac{-2/3}{2}=-\frac{2}{6}=-\frac{1}{3}$$

$\Rightarrow \frac{-1}{c^2}=-\frac{1}{3} \Rightarrow c=\pm\sqrt{3}$ only $\sqrt{3}\in[1,3]$

Q4: Show that the equation $2x+\cos(x)=0$ has exactly one real root.

$2x$ cont it is a polynomall

$\cos x$ cont trigonometric sin & cos cont. in all $\mathbb{R}$

$\Rightarrow f(x)=2x+\cos(x)$ is cont in all $\mathbb{R}$

Now $f(0)=2(0)+\cos 0=1>0$

$f(-\frac{\pi}{2})=-\pi+\cos\frac{-\pi}{2}=-\pi<0$

by IMVT $\Rightarrow \exists c\in\mathbb{R}$ such that

$f(c)=0 \Rightarrow f(x)$ has solution

or $2x+\cos(x)=0$ has a root

Now $f'(x)=2-\sin x>0$

$\Rightarrow f(x)$ is incresing function

for all Real number $\Rightarrow$ it will cross the x-axis only one time